蔡氏电路的混沌数值模拟

近代物理实验Ⅰ　周四　B3组

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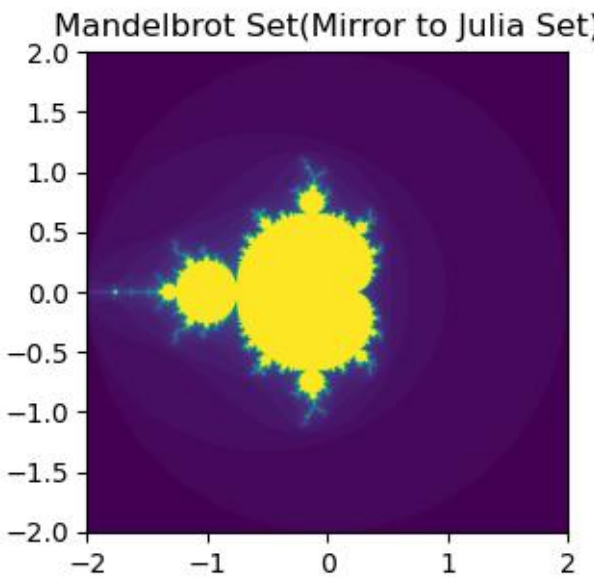
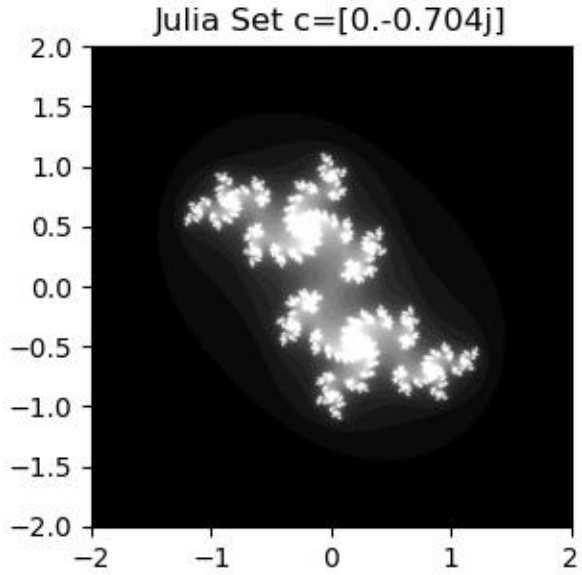
混沌

1、对初始条件的敏感性——“蝴蝶效应”

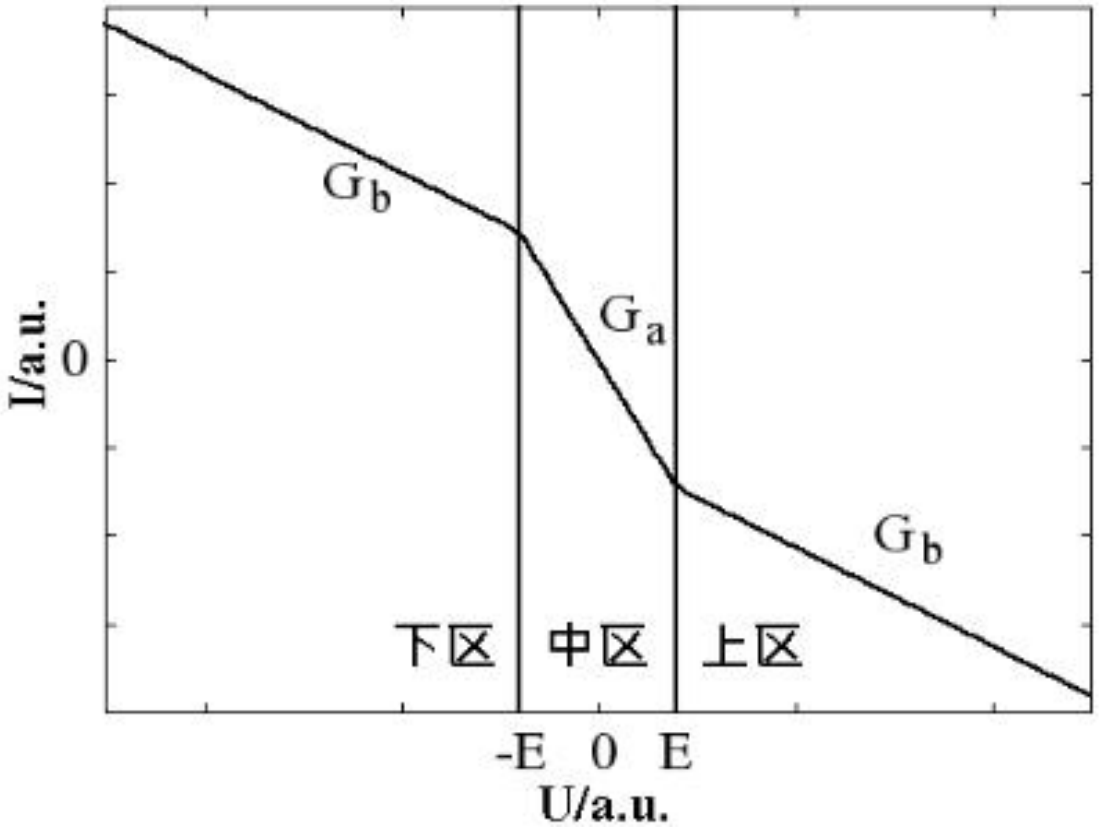
2、分形

3、奇异吸引子





蔡氏电路



$$\begin{cases} C_1 \dfrac{dU_1}{dt} = G(U_2 - U_1) - g(U_1) \\ C_2 \dfrac{dU_2}{dt} = G(U_1 - U_2) + I_L \\ L \dfrac{dI_L}{dt} = -U_2 \end{cases}$$

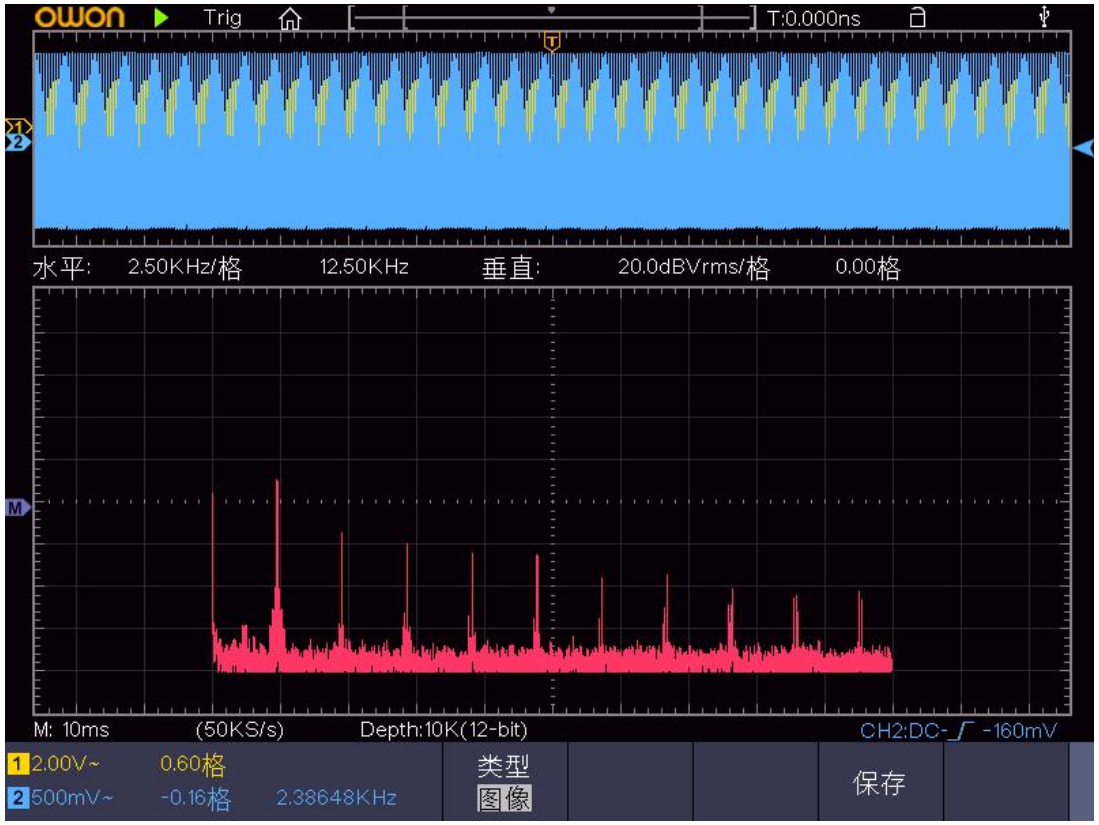
元件参数测量

- LCR meter的测量结果与频率有关

频率/Hz	C1/nF	C2/nF	L/mH
100	11.10	101.802	29.1802
1k	11.0381	100.674	23.7095
2k	11.061	101.00	23.3130
5k	11.0151	100.155	23.0538
10k	10.8756	99.5024	22.9948

混沌相图的模拟

- 模拟参数的选取 (2kHz)



- 电路元件参数

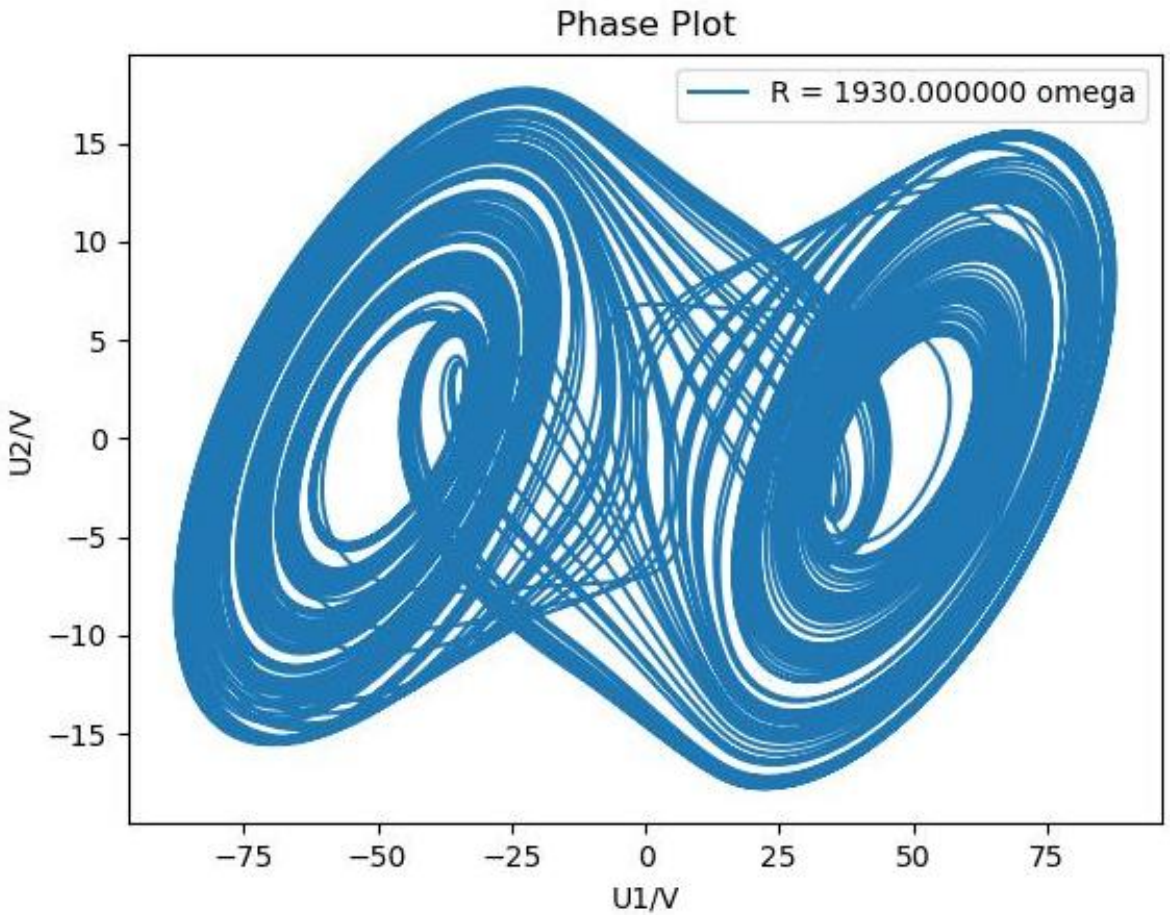
```python
# constants
global C1, C2, L, R, G
C1 = 11.061e-9 # F
C2 = 101.0e-9 # F
L = 23.7095e-3 # H
R = 2040# omega
G = 1/R
```

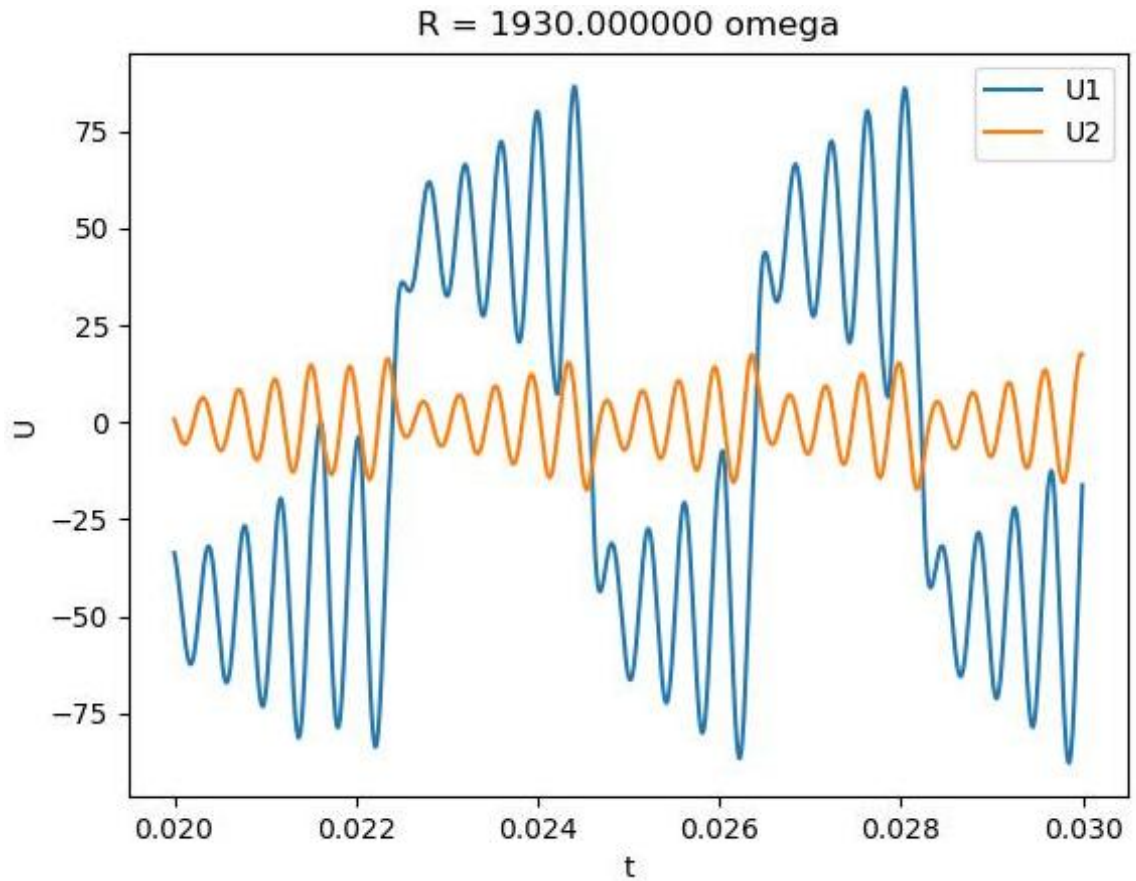
```python
def g(U):
    Ga = -0.00076 # omega^(-1) uncertainty: 0.1*10^(-4)
    Gb = -0.000409 # omega^(-1) uncertainty: 0.06*10^(-4)
    E = 15.0 #V
    g = Gb*U+(Gb-Ga)/2*(abs(U-E)-abs(U+E))
    return g
```

- 方程初始条件

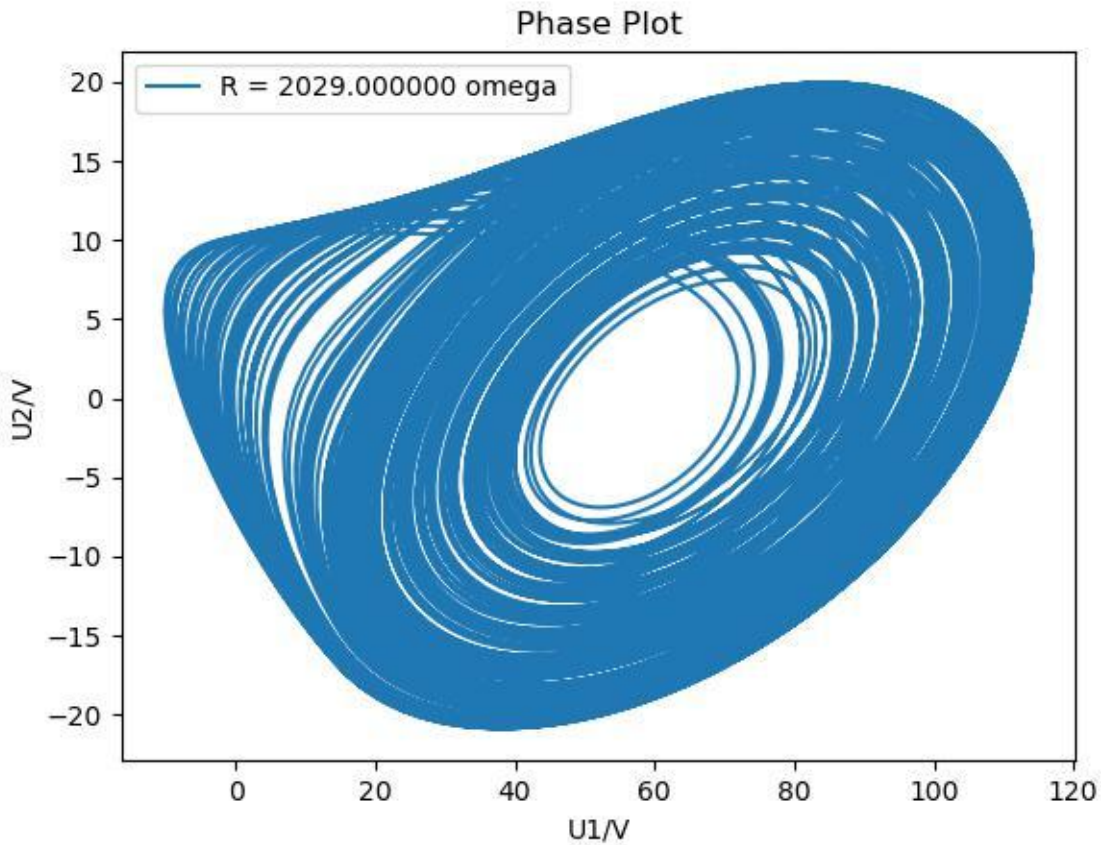
```python
# initial condition
t0 = 0
y0 = np.array([0.0, 0.0, 0.0001]) # [U1, U2, I]
dt = 0.1
t_final = 20000
```

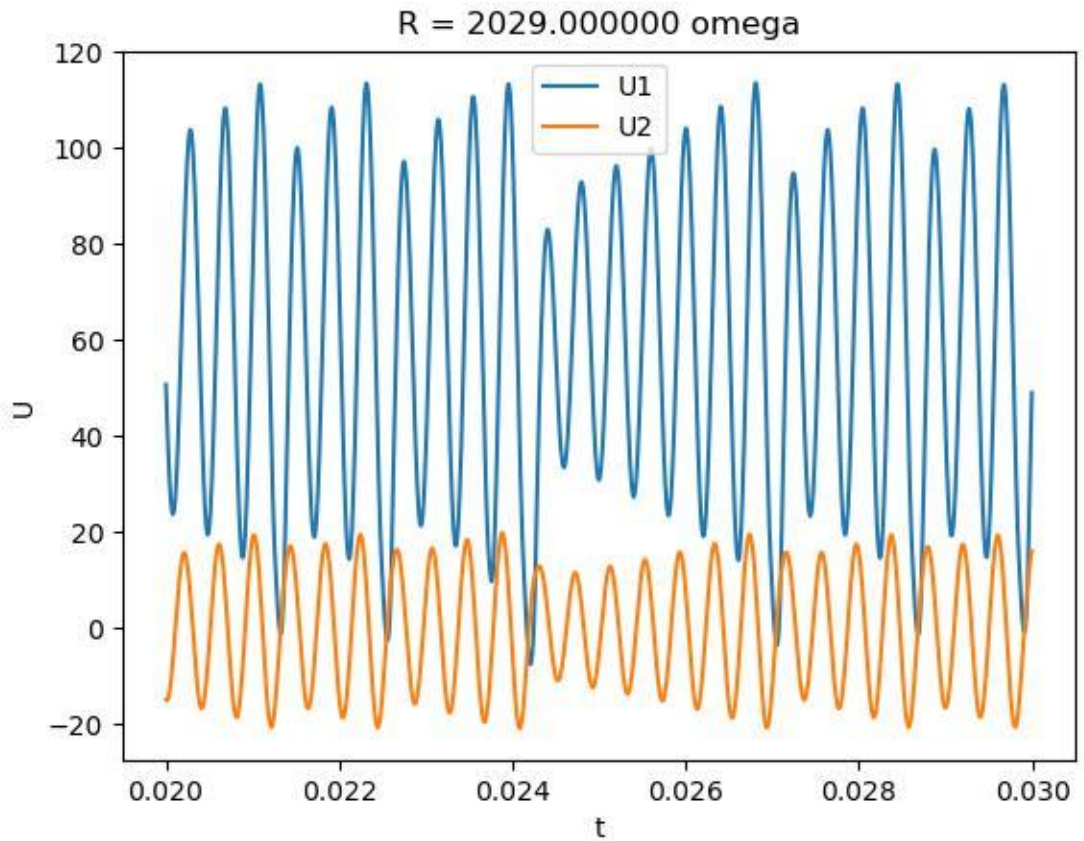
• 双吸引子


Phase Plot
R = 1930.000000 omega
U2/V
U1/V
15
10
5
0
−5
−10
−15
−75
−50
−25
0
25
50
75

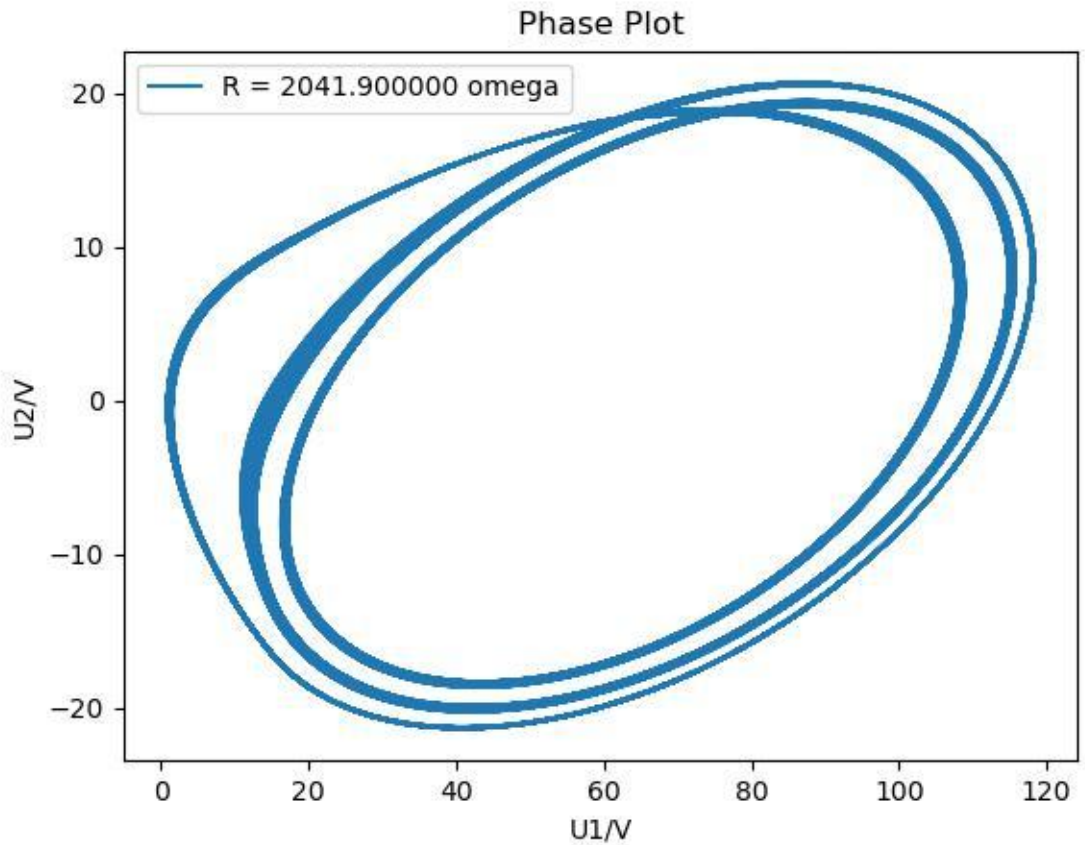

R = 1930.000000 omega
U1
U2
U
t
75
50
25
0
−25
−50
−75
0.020
0.022
0.024
0.026
0.028
0.030

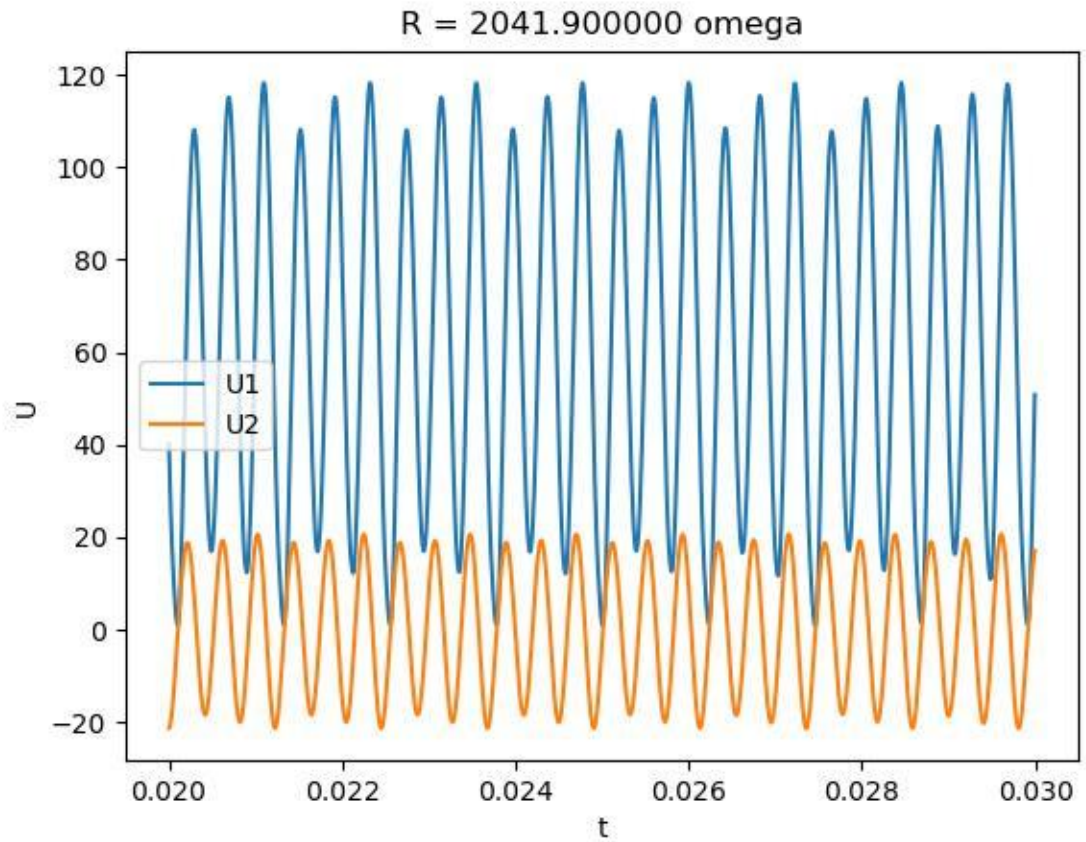
- 单吸引子


Phase Plot
R = 2029.000000 omega
U2/V
U1/V
20
15
10
5
0
−5
−10
−15
−20
0
20
40
60
80
100
120

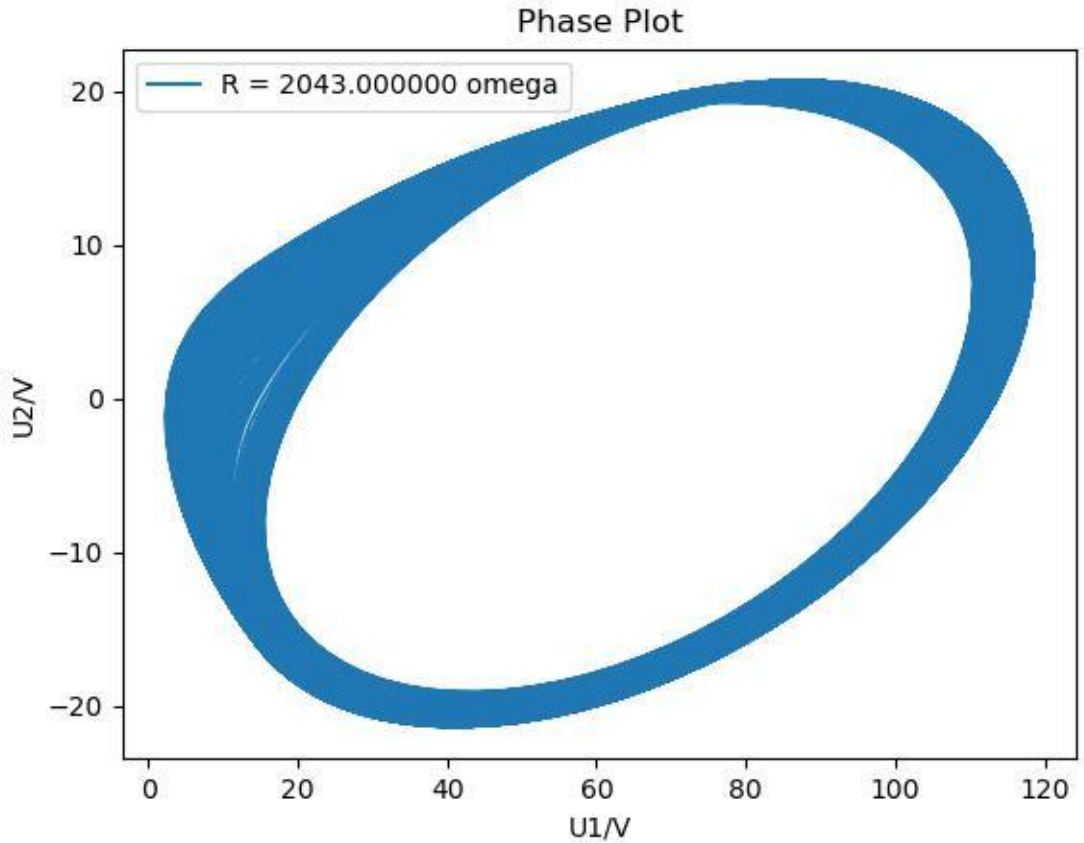

R = 2029.000000 omega
U1
U2
U
t
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

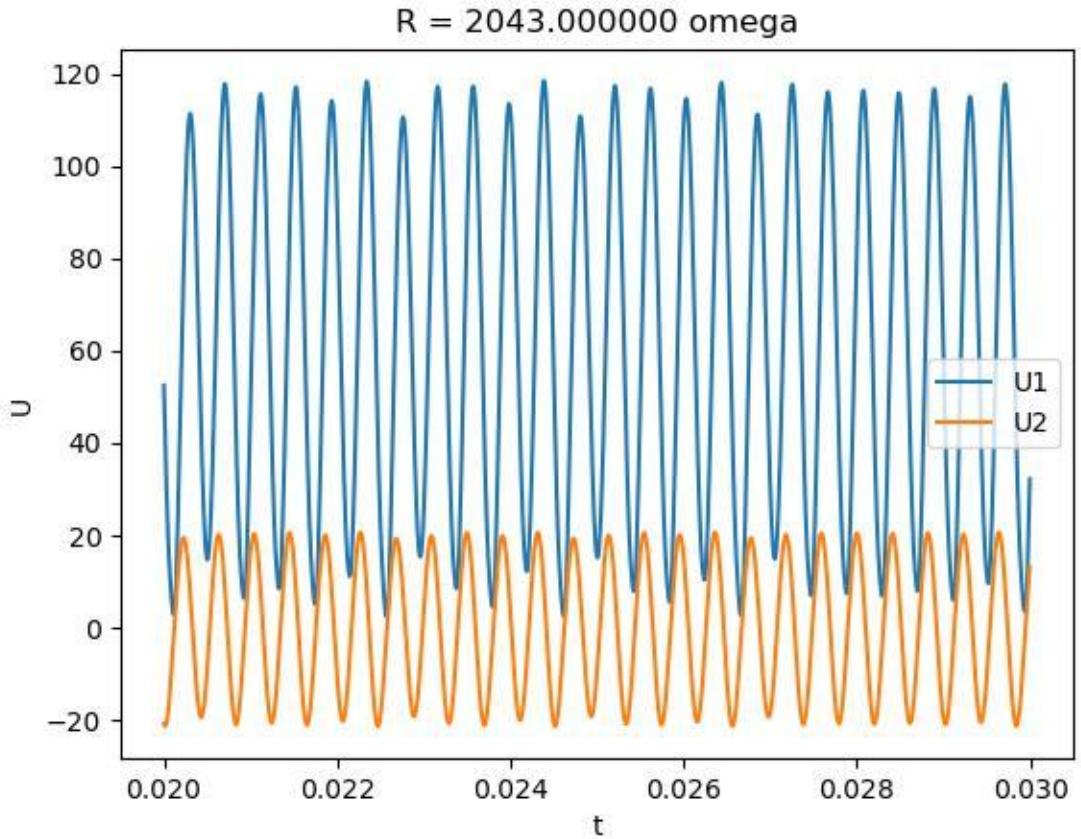
- 三周期


Phase Plot
R = 2041.900000 omega
U2/V
U1/V
20
10
0
−10
−20
0
20
40
60
80
100
120

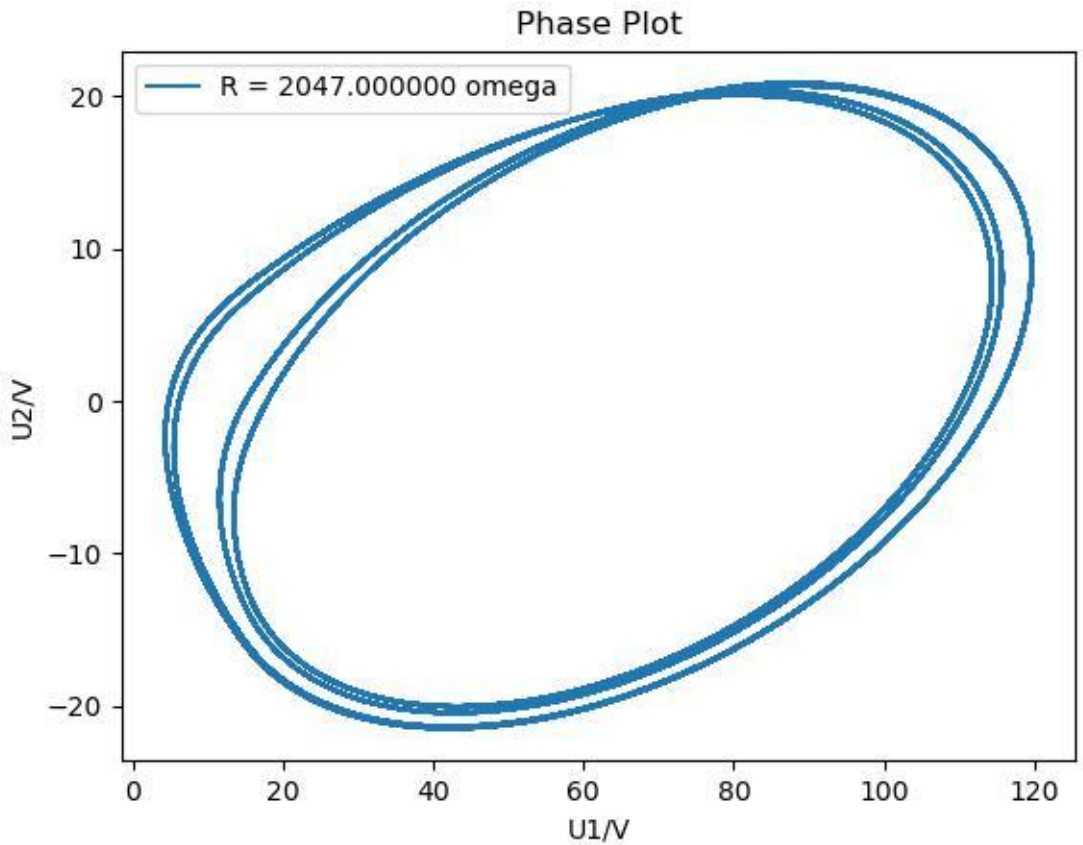

R = 2041.900000 omega
U1
U2
U
t
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

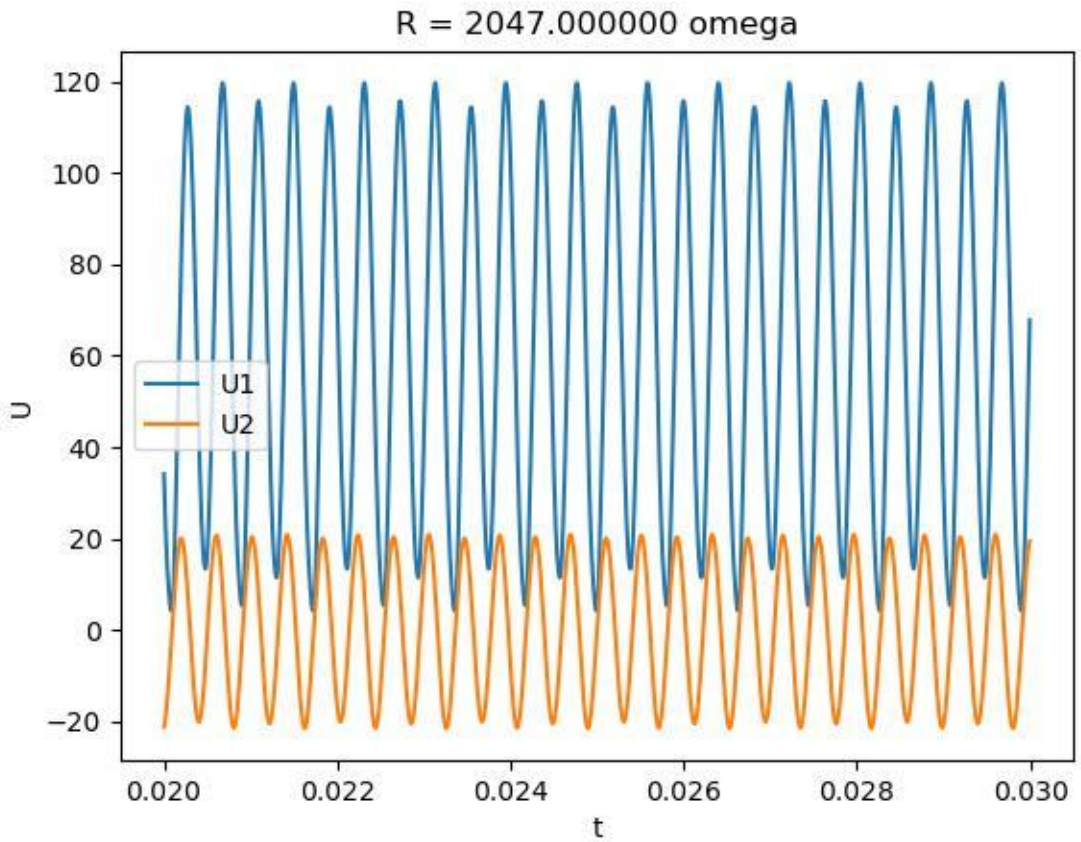
- 阵发混沌


Phase Plot
R = 2043.000000 omega
U2/V
U1/V
20
10
0
−10
−20
0
20
40
60
80
100
120

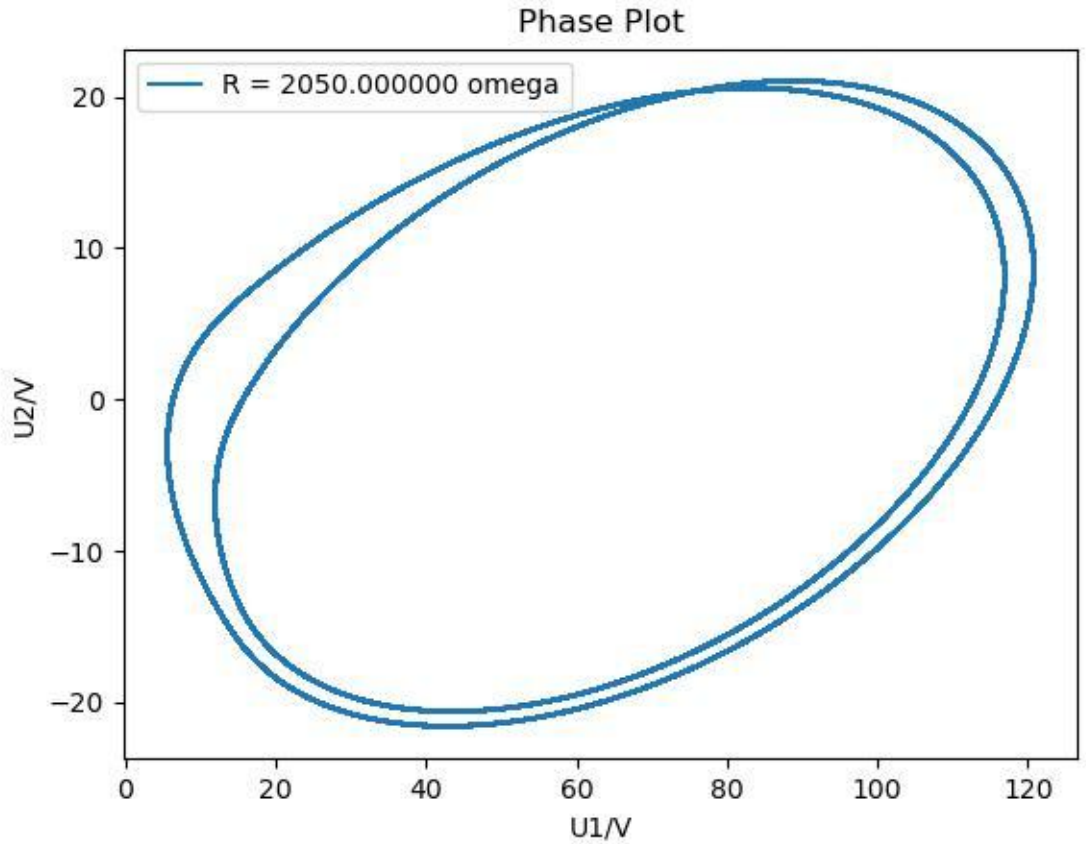

R = 2043.000000 omega
U
t
U1
U2
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

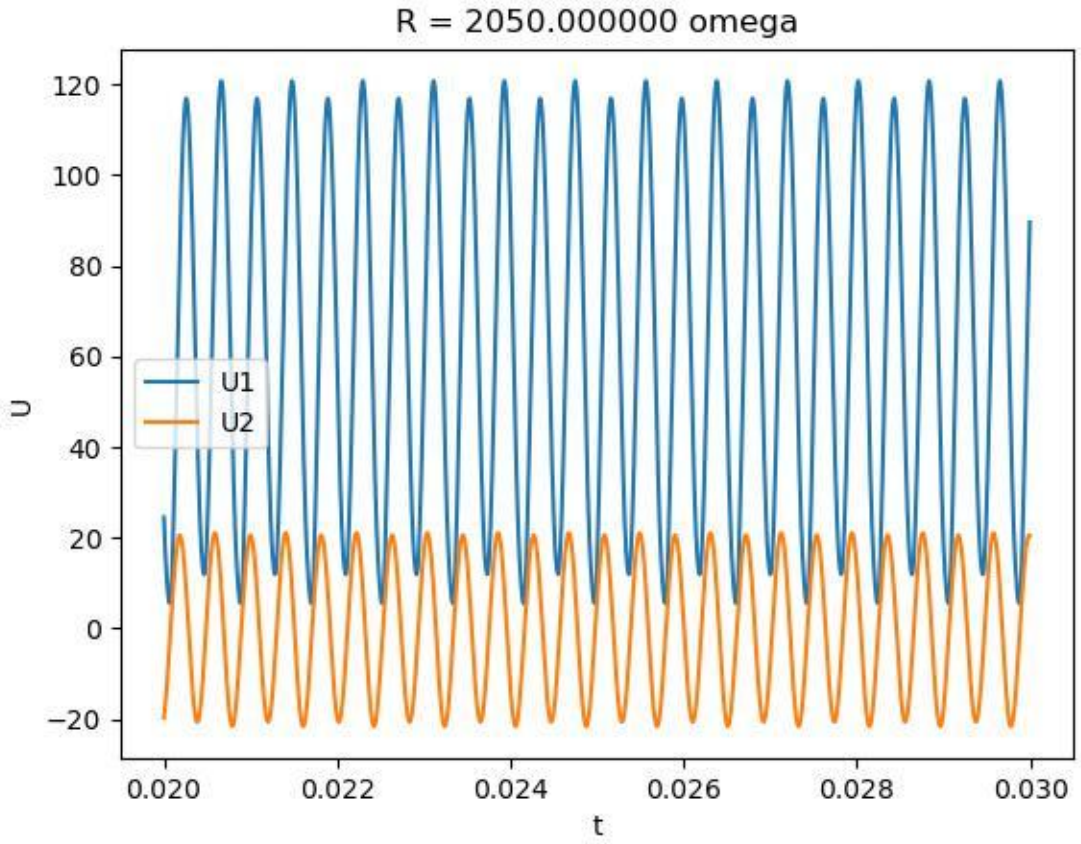
- 四周期


Phase Plot
R = 2047.000000 omega
U2/V
U1/V
20
10
0
−10
−20
0
20
40
60
80
100
120

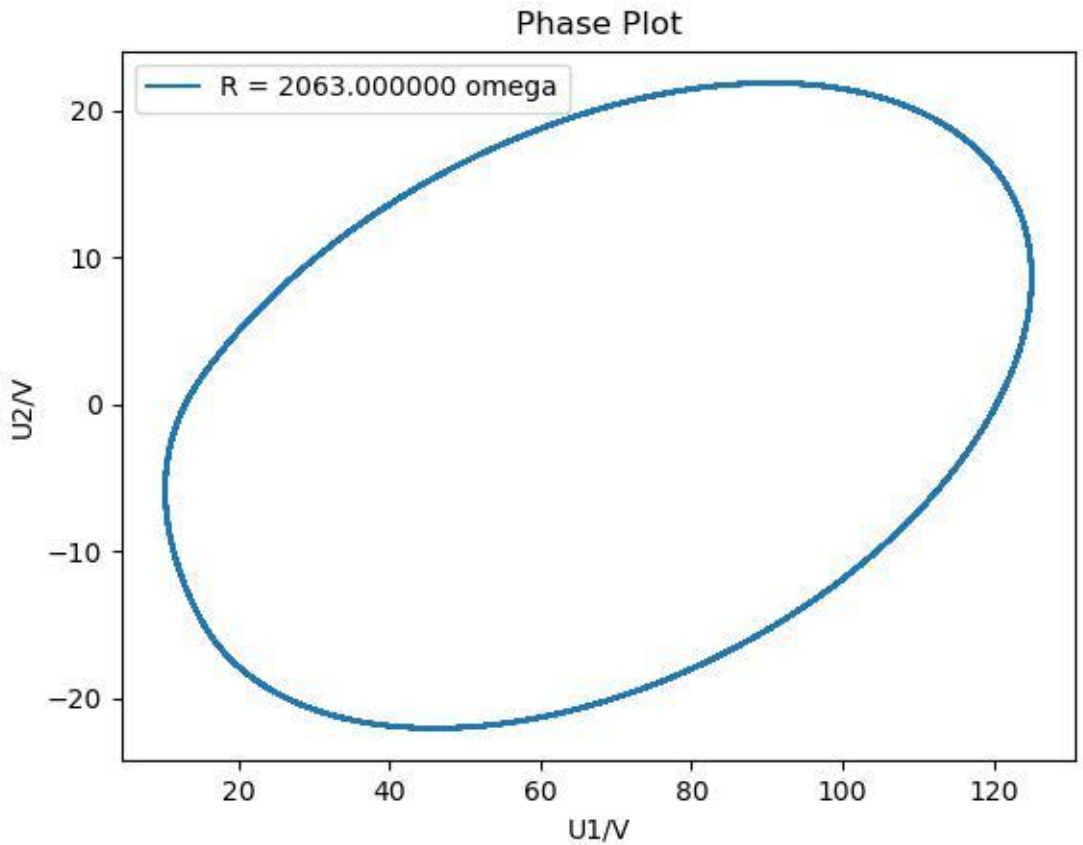

R = 2047.000000 omega
U1
U2
U
t
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

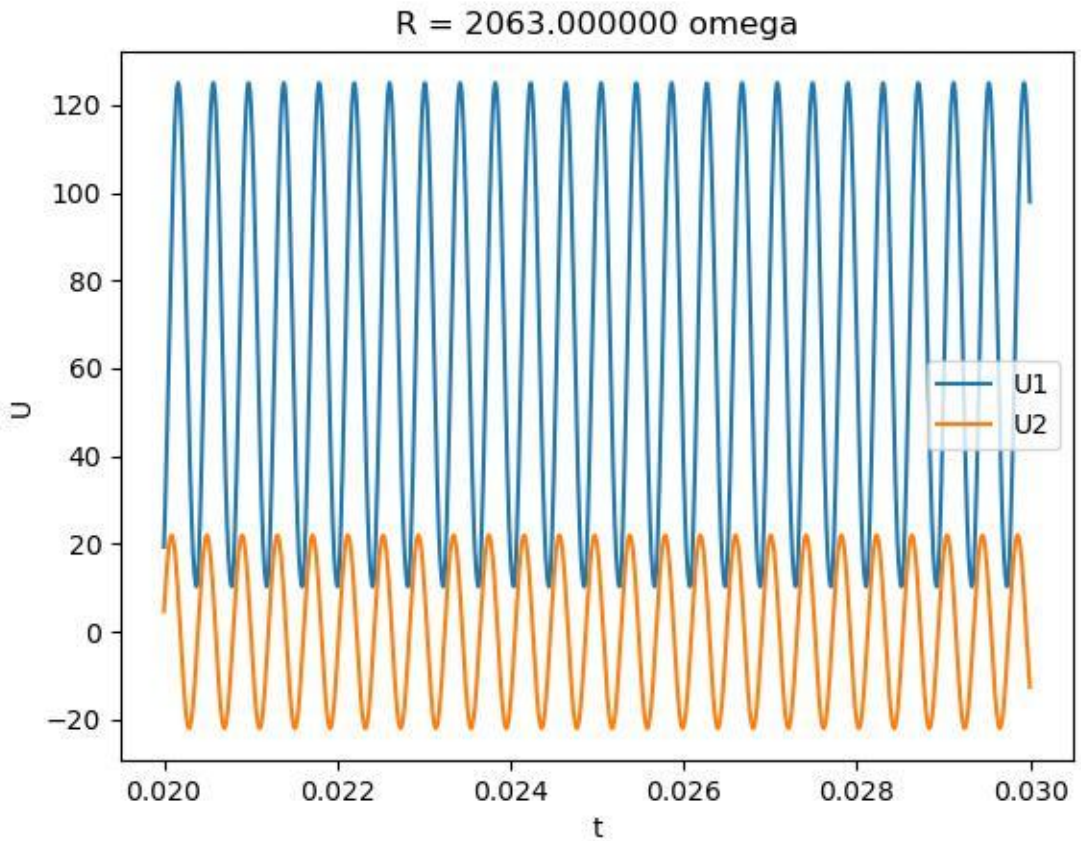
- 双周期


Phase Plot
R = 2050.000000 omega
U2/V
U1/V
20
10
0
−10
−20
0
20
40
60
80
100
120


R = 2050.000000 omega
U
t
U1
U2
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

- 单周期

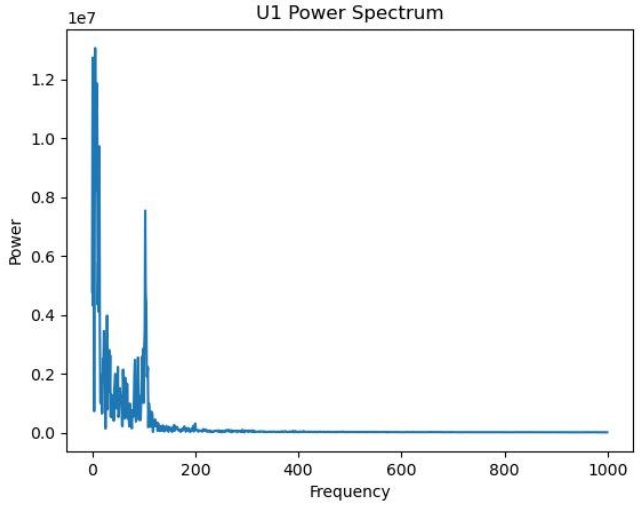

Phase Plot
R = 2063.000000 omega
U2/V
U1/V
20
10
0
−10
−20
20
40
60
80
100
120

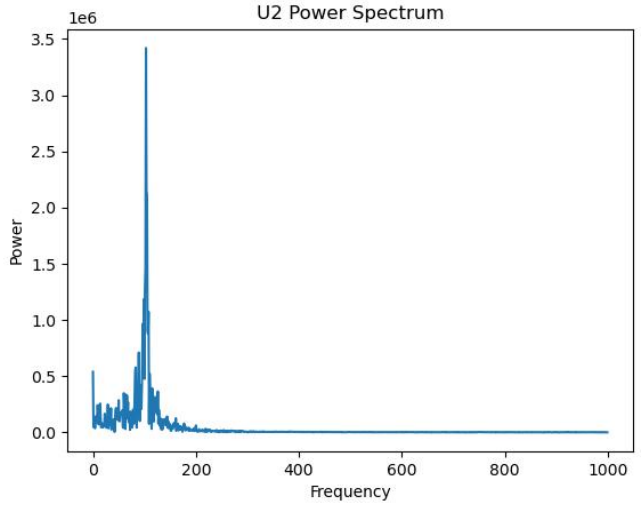

R = 2063.000000 omega
U
t
U1
U2
120
100
80
60
40
20
0
−20
0.020
0.022
0.024
0.026
0.028
0.030

FFT分析实验和模拟数据

- 双吸引子

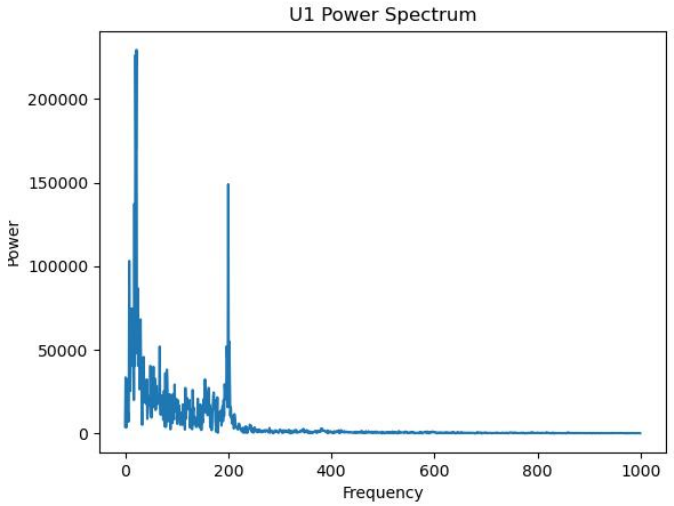
实验数据

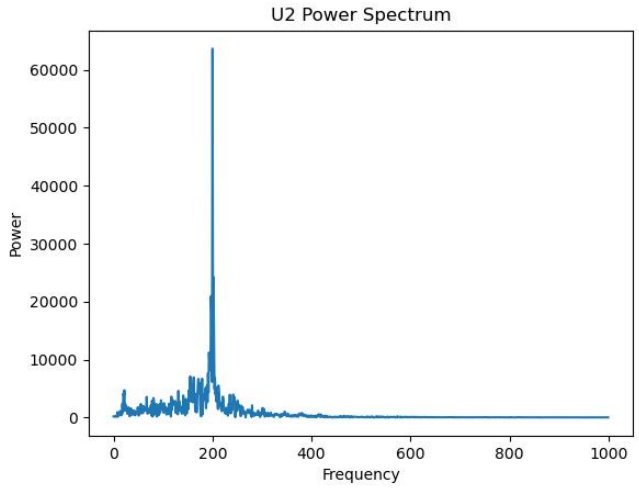


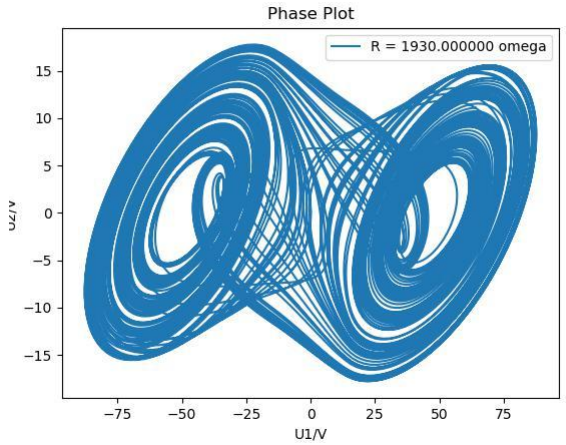


频率分布形状相同，右侧峰对应频率不同的原因是时间采样序列不同！

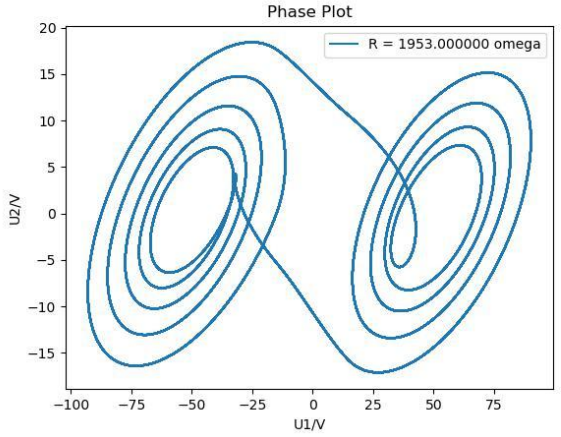
模拟数据





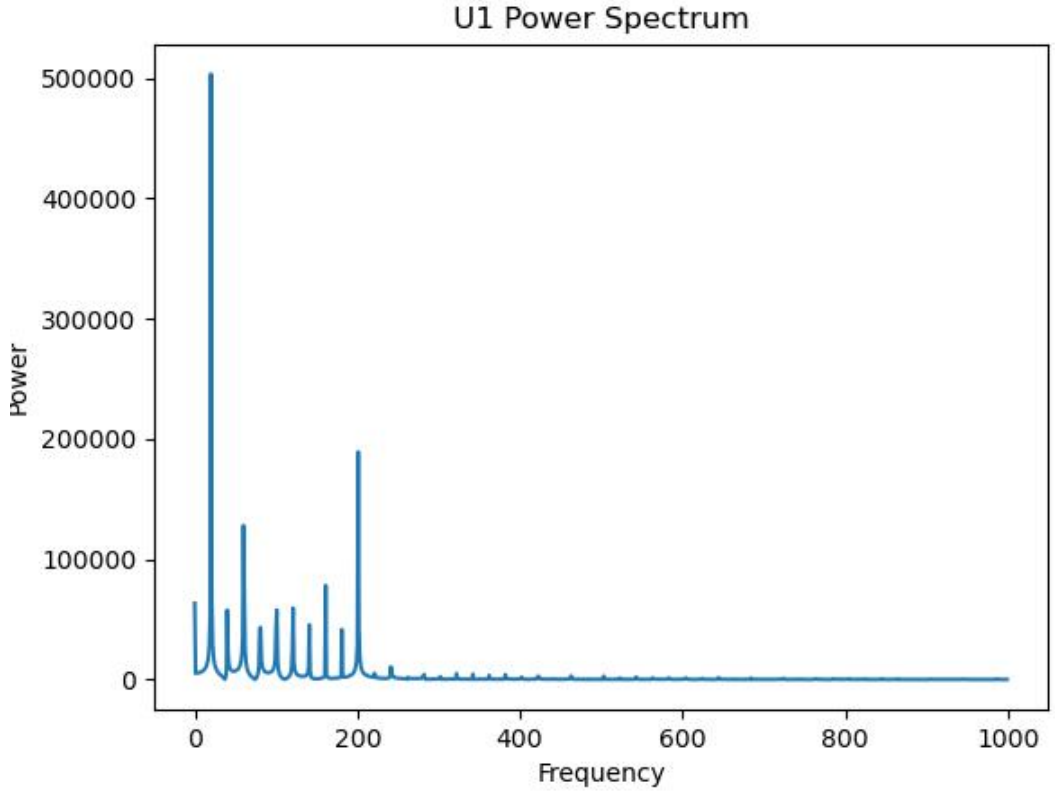


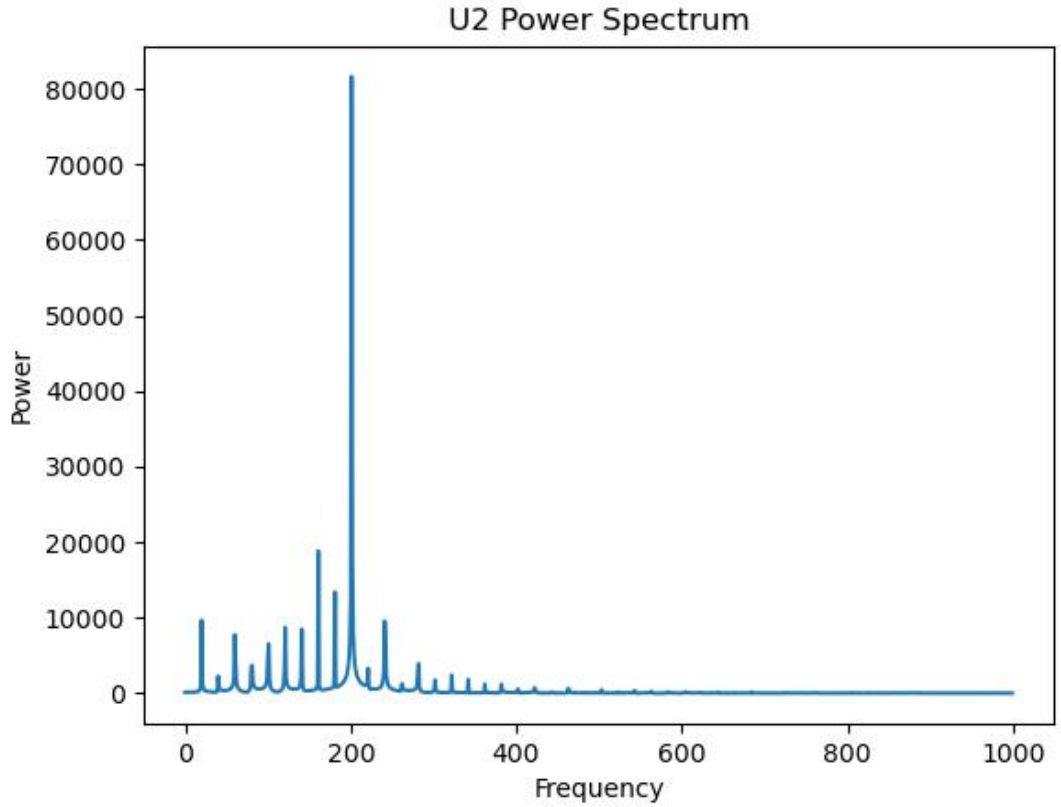
- 双吸引子



左侧5个圈，右侧4个圈，对应功率谱值200处峰前的9个峰

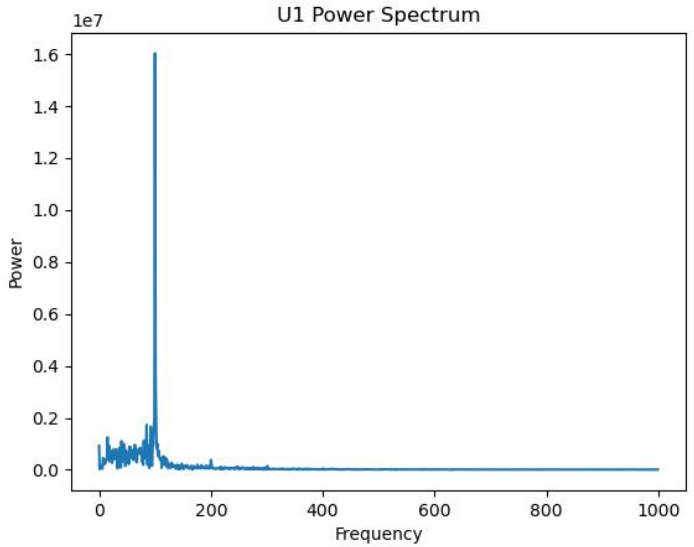
U1第一个频率高的原因：遍历了双吸引子的整周期，U2没有

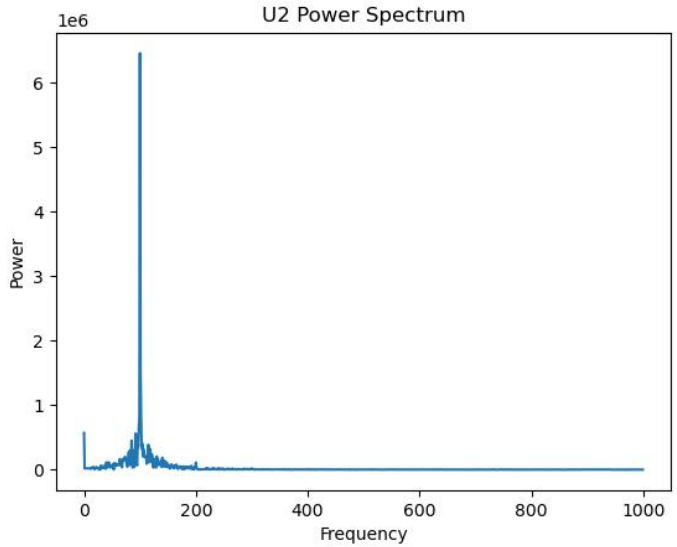




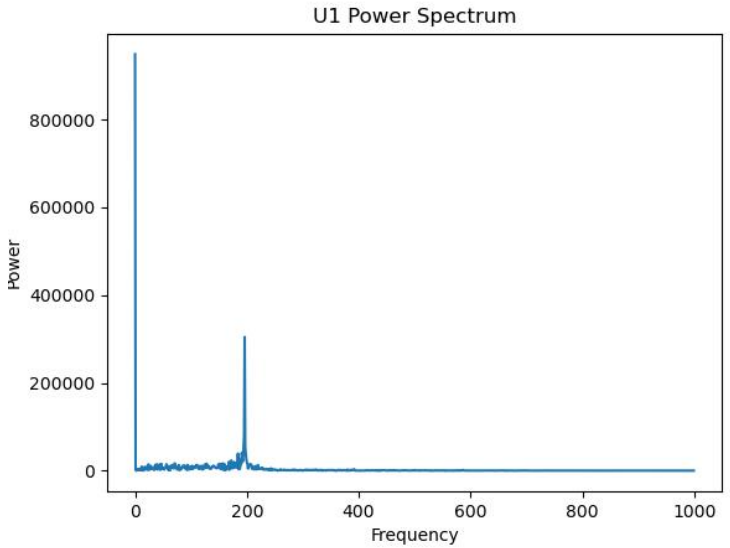
- 单吸引子

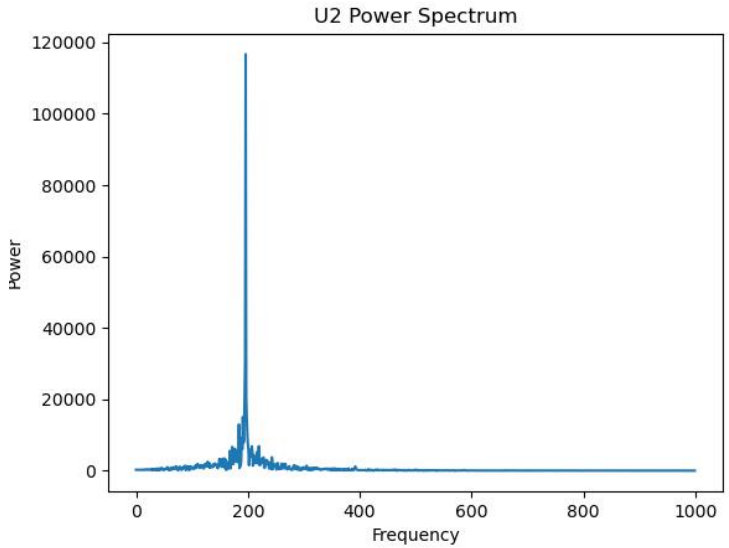
实验数据

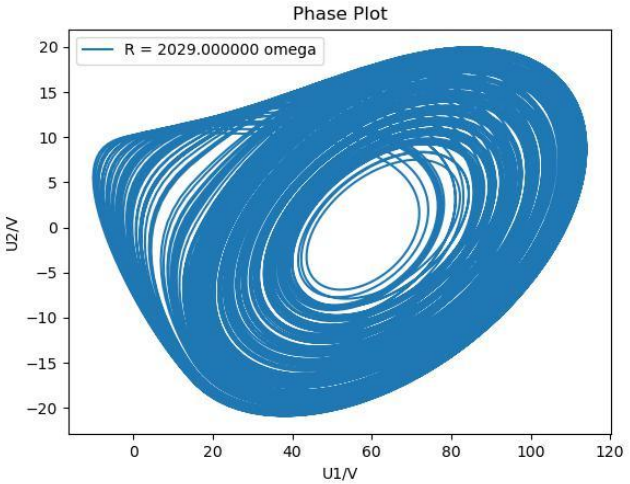




模拟数据

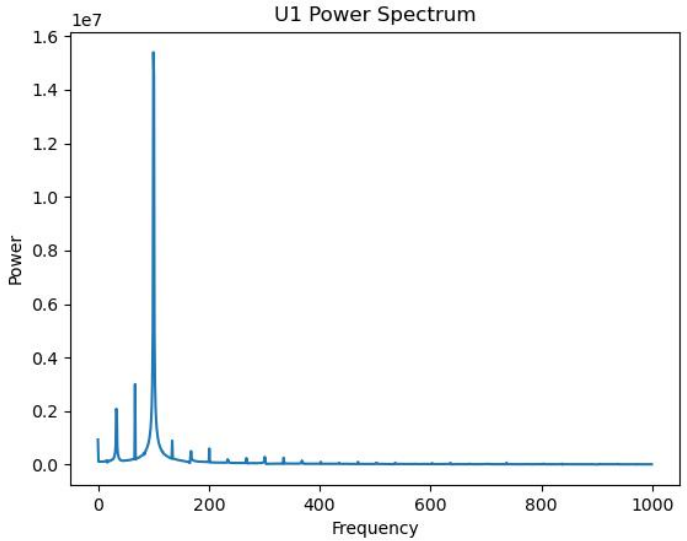


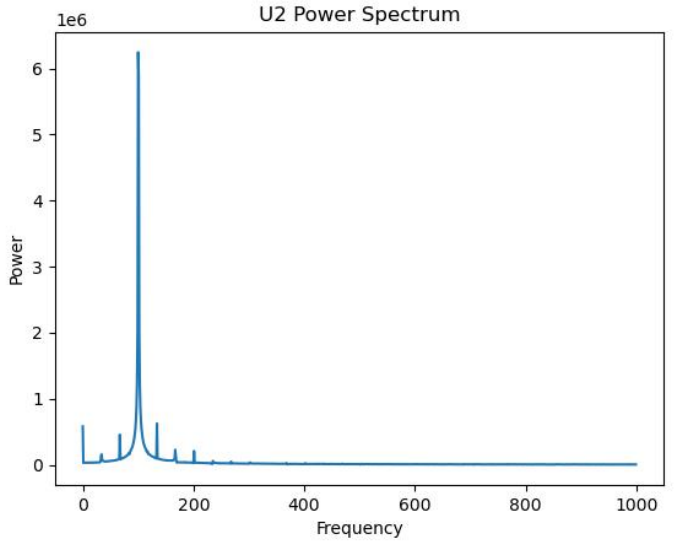




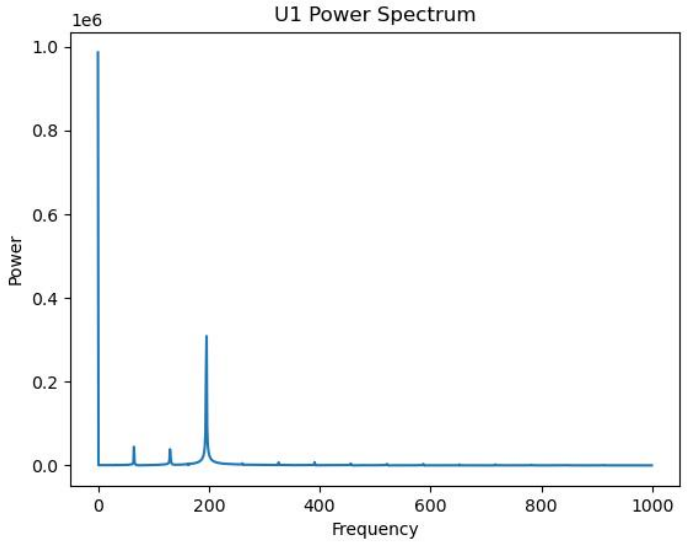
- 三周期

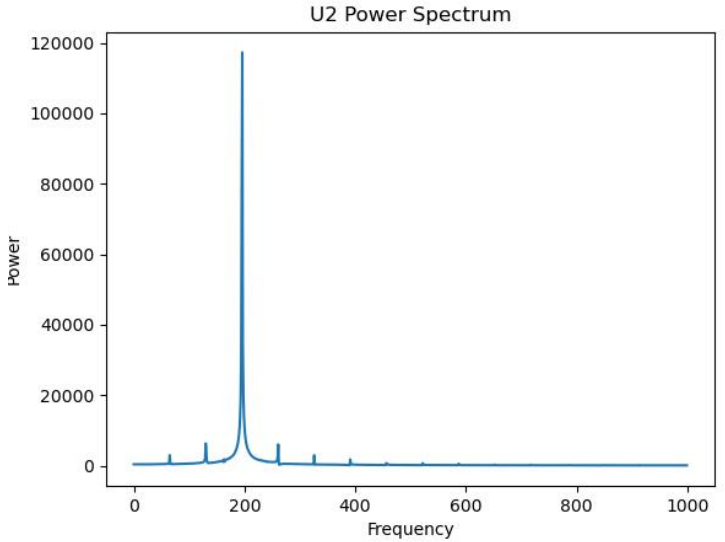
实验数据

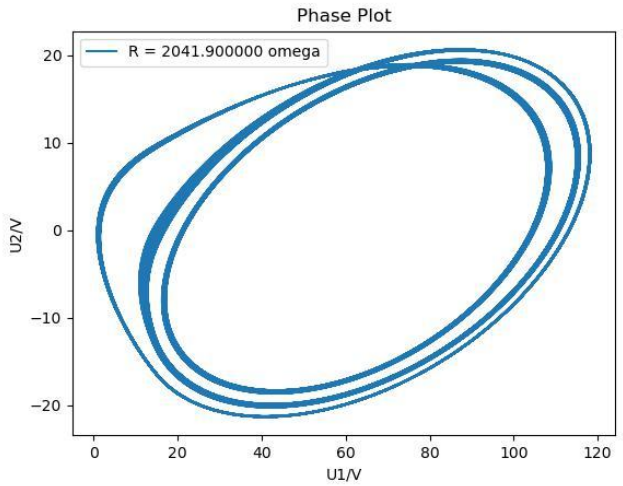




模拟数据

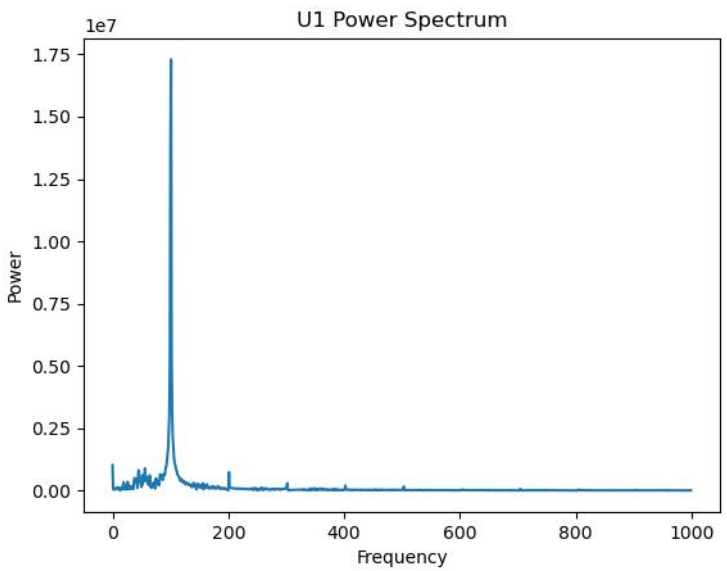


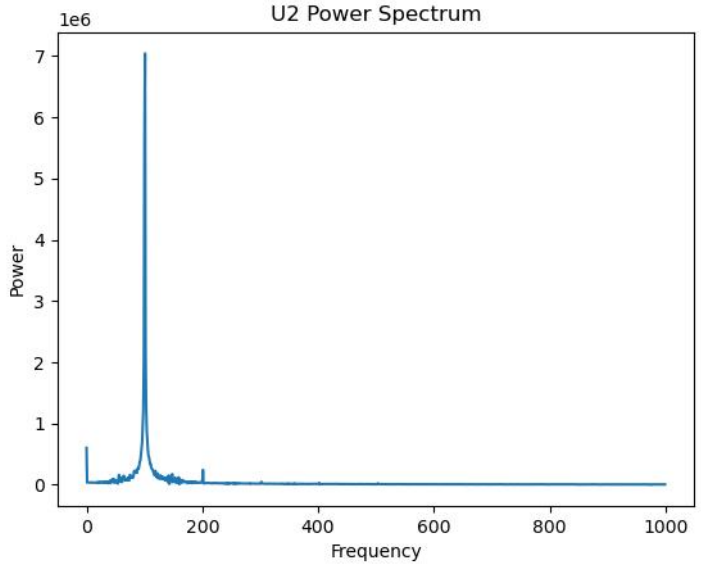




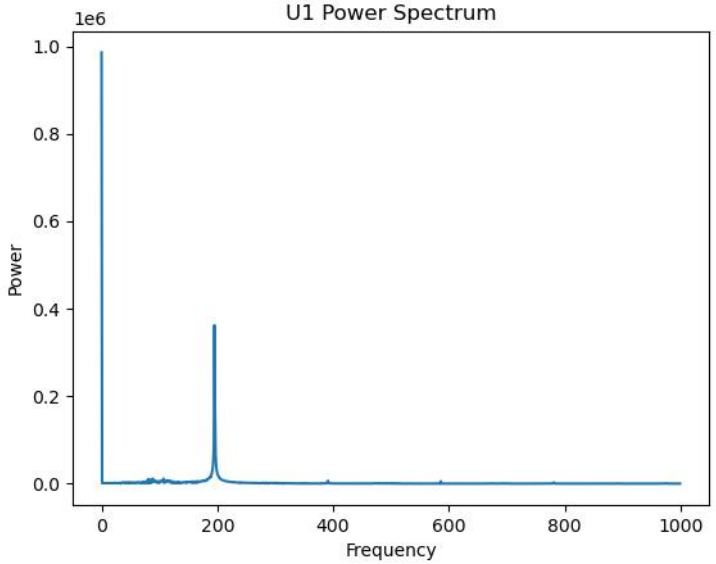
- 阵发混沌

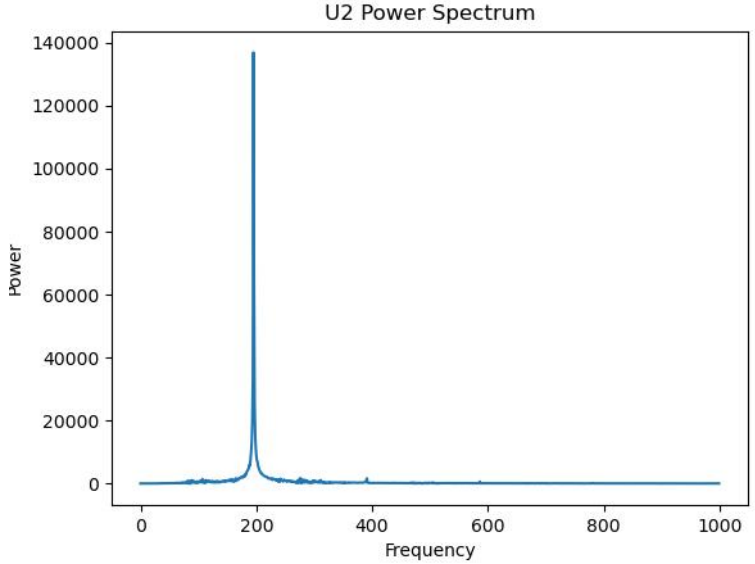
实验数据

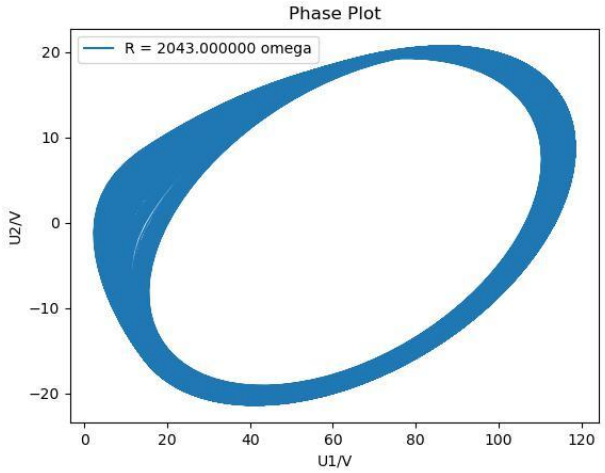




模拟数据

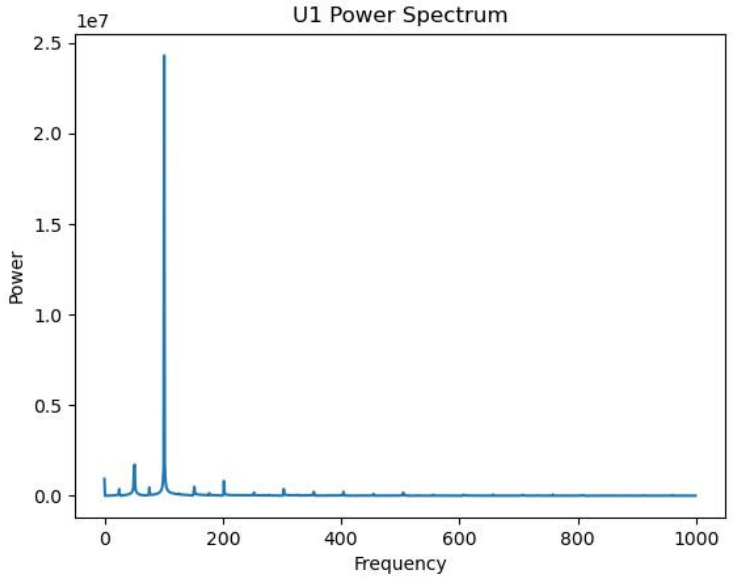


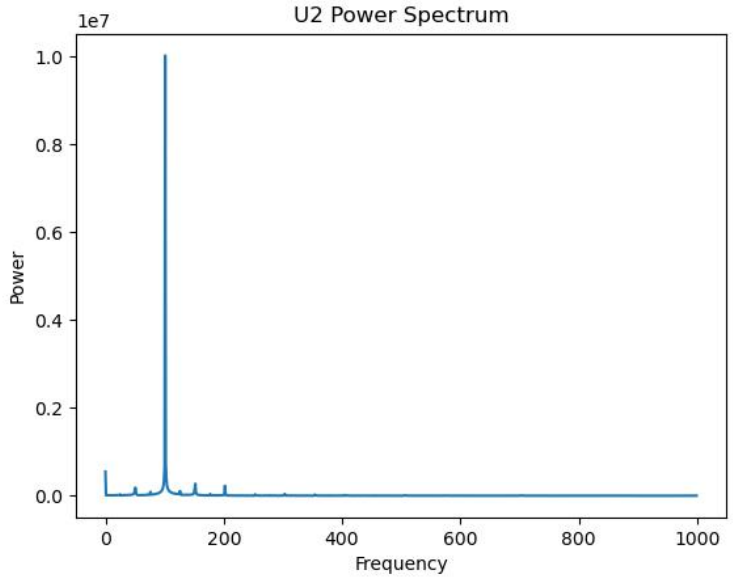




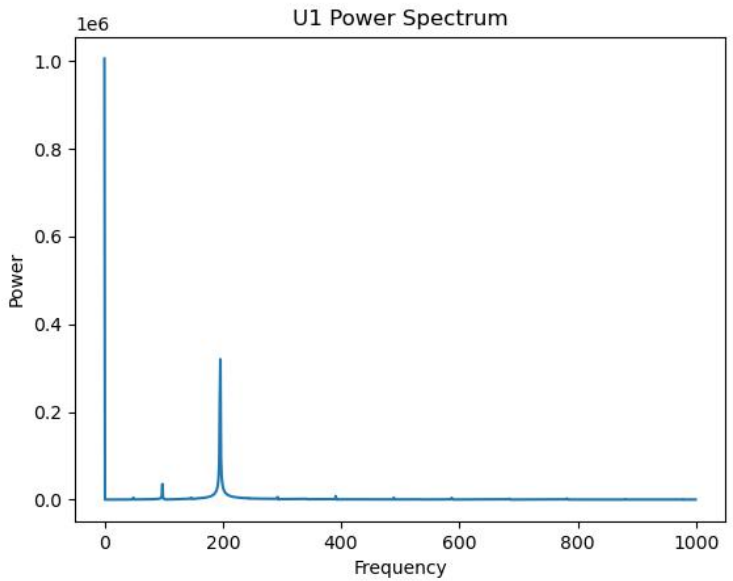
- 四周期

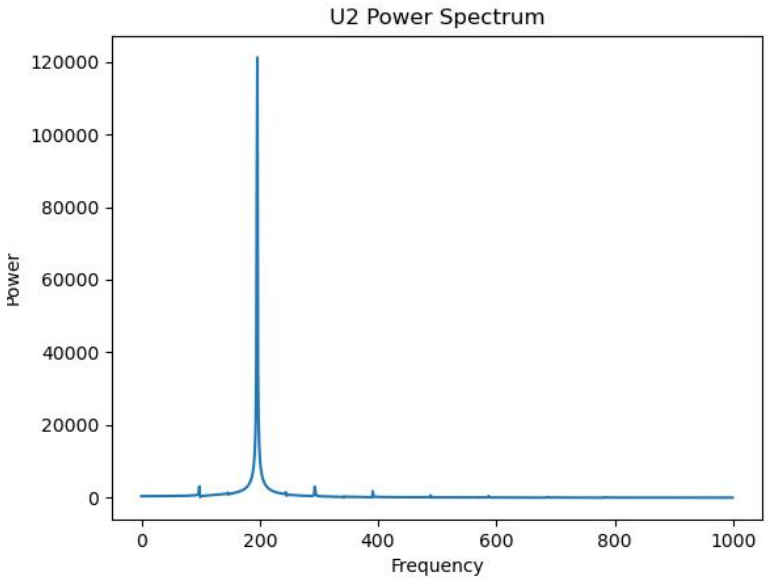
实验数据

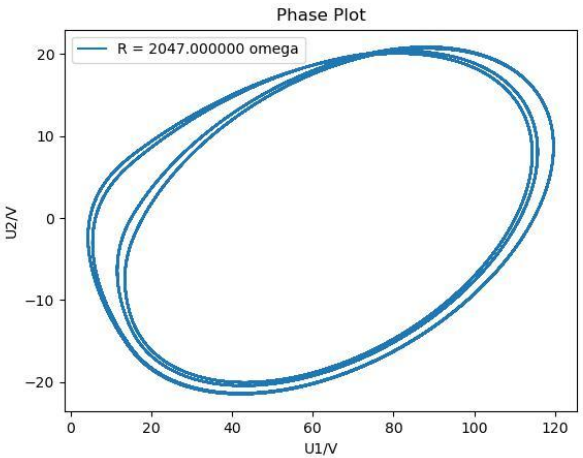




模拟数据

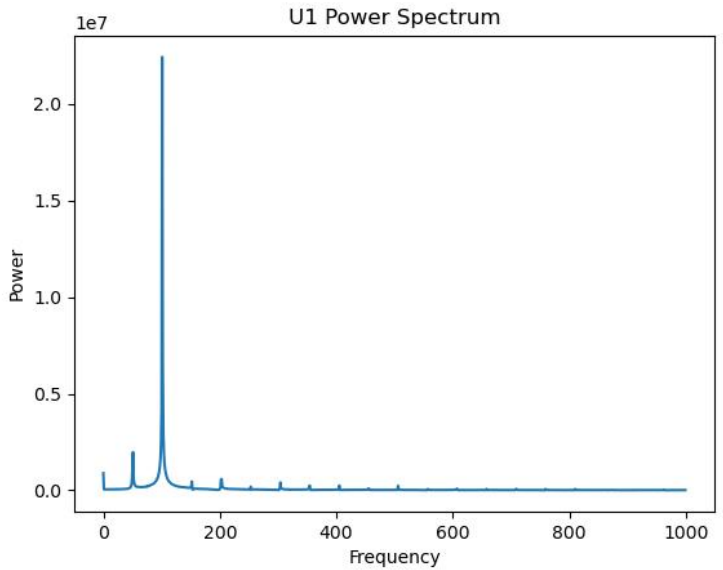


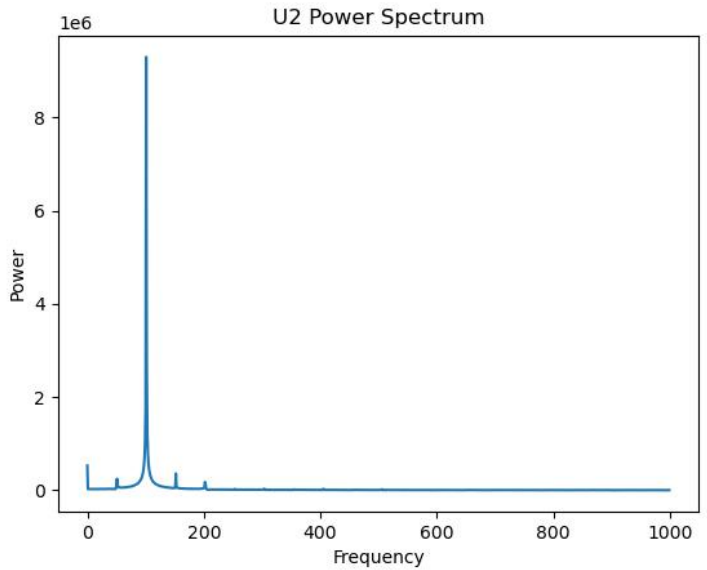




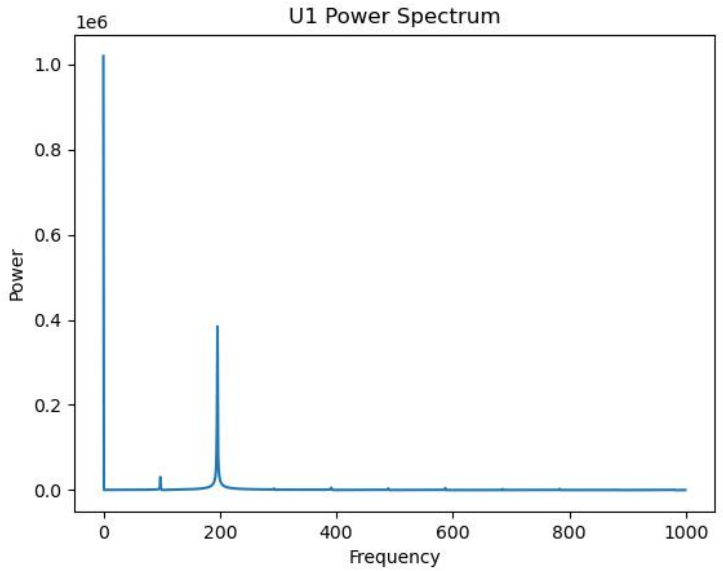
- 双周期

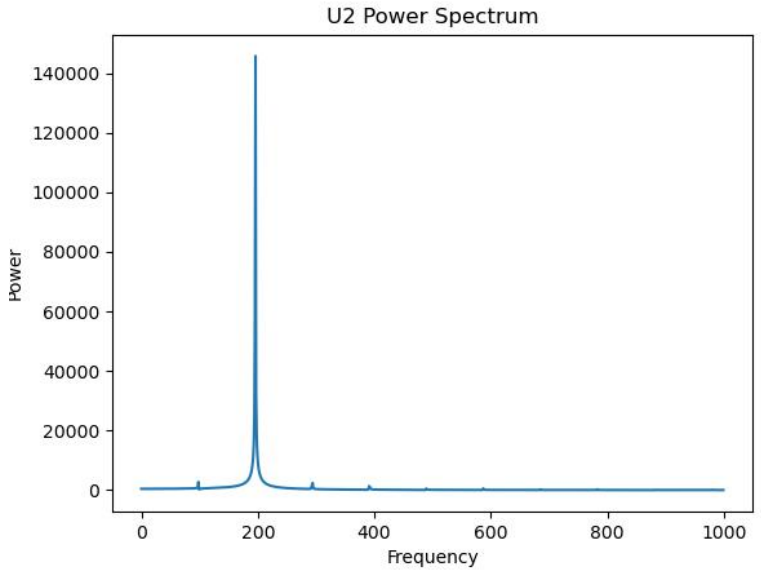
实验数据

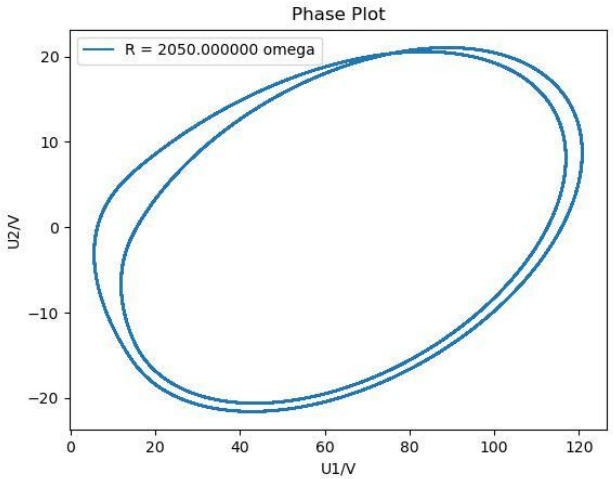




模拟数据

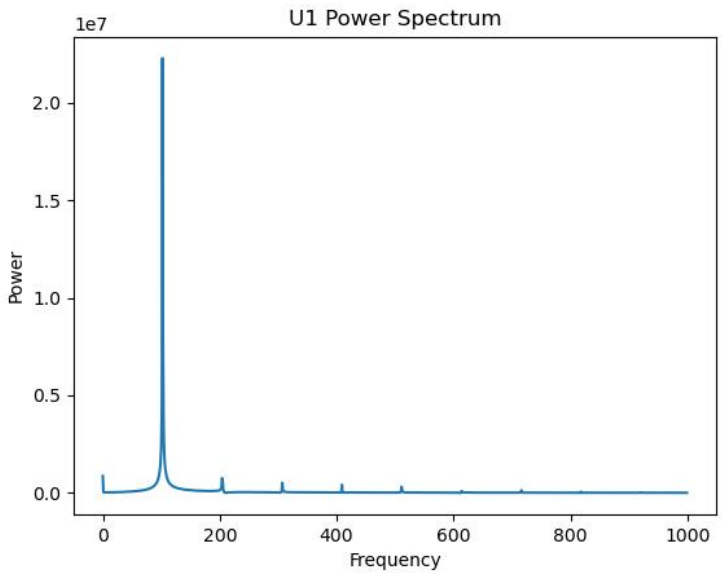


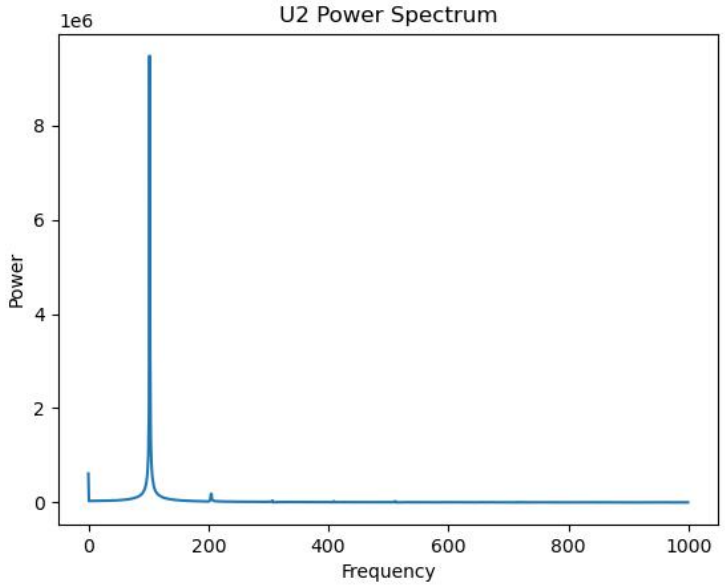




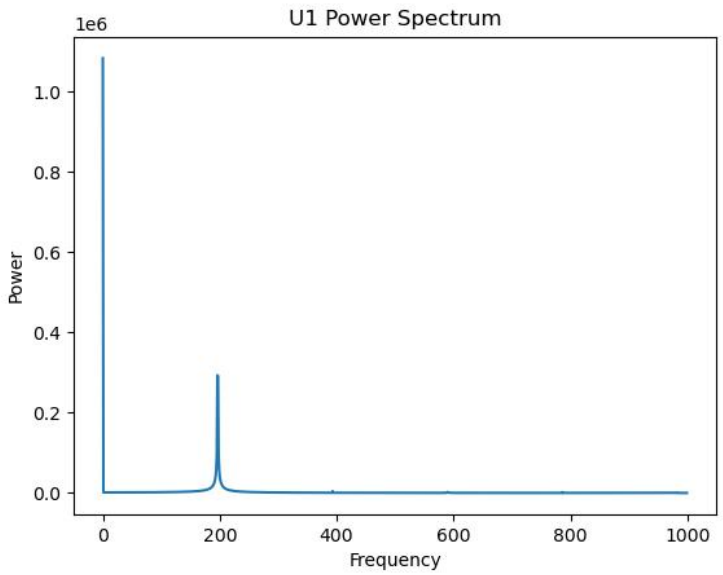
- **单周期**

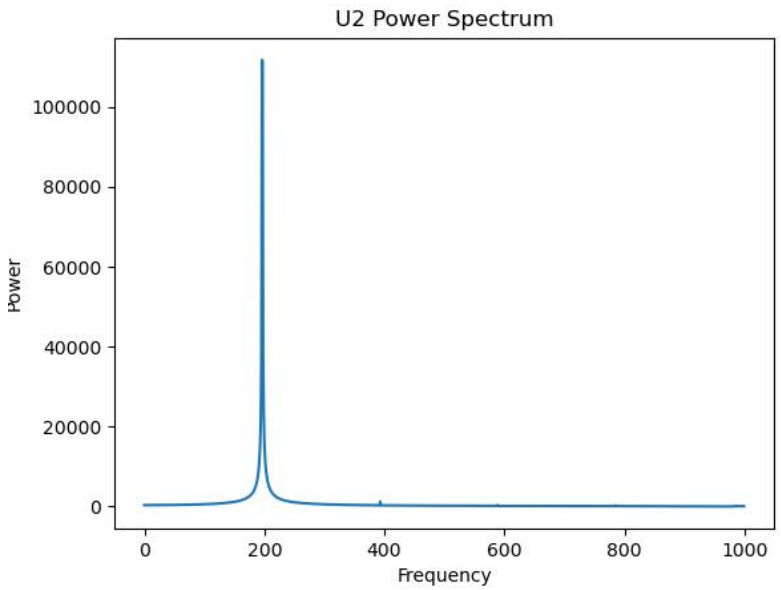
实验数据

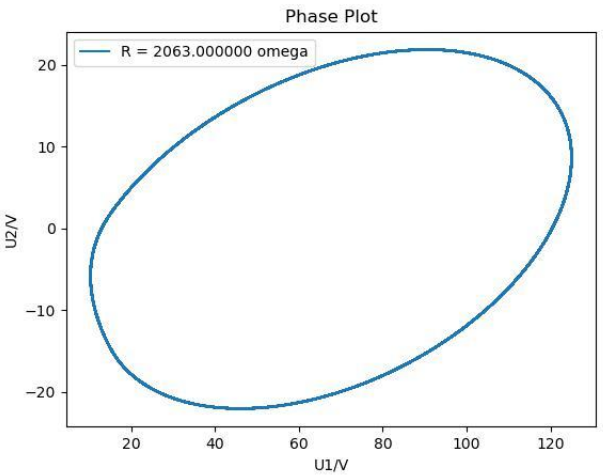




模拟数据

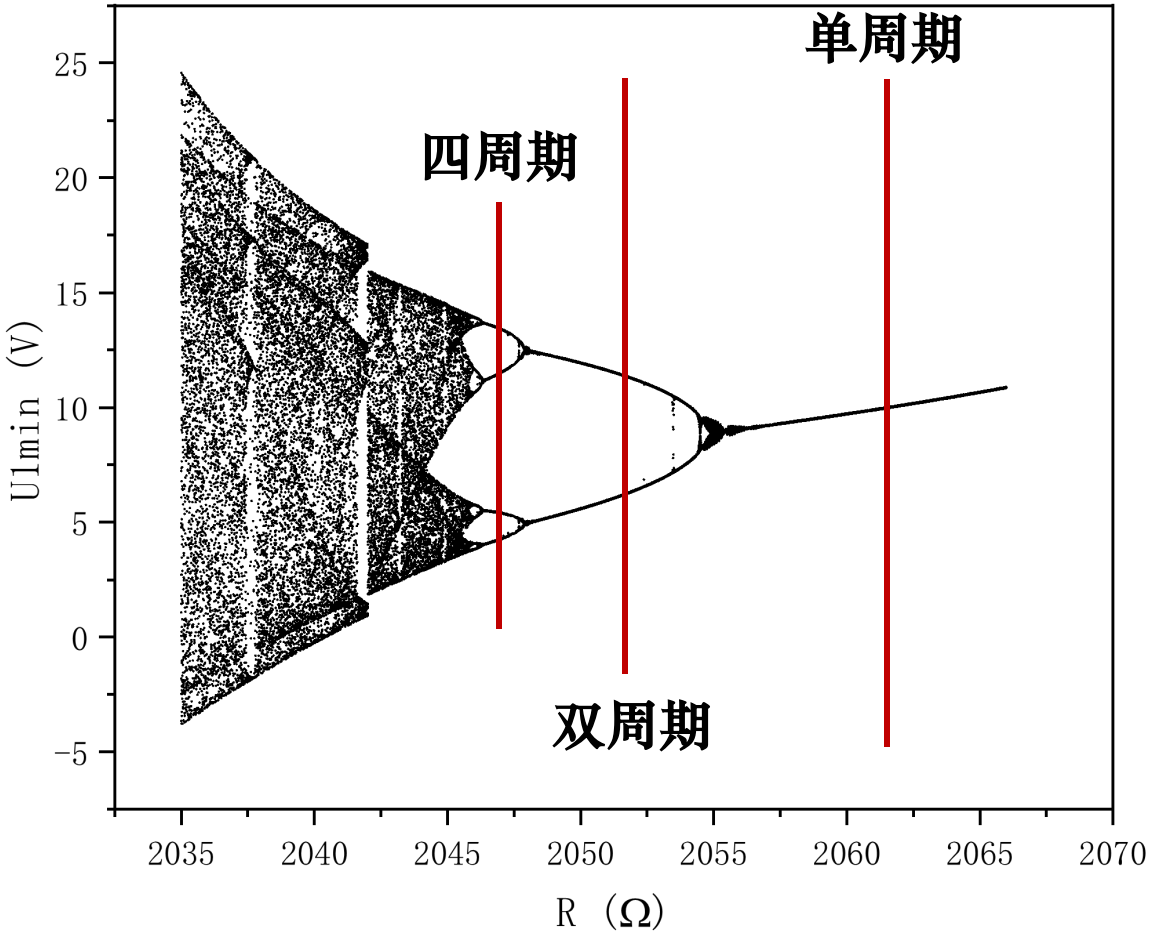




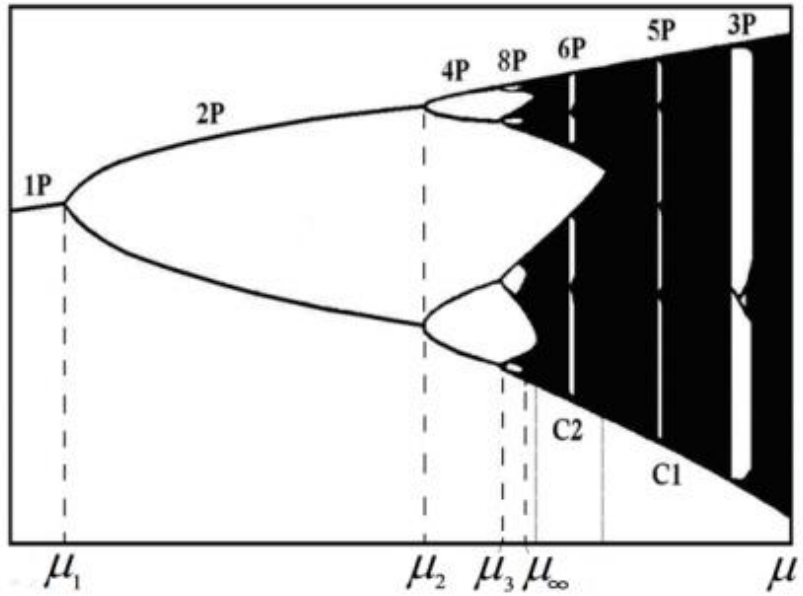


分岔图

- 倍周期分岔



- 费根鲍姆常数

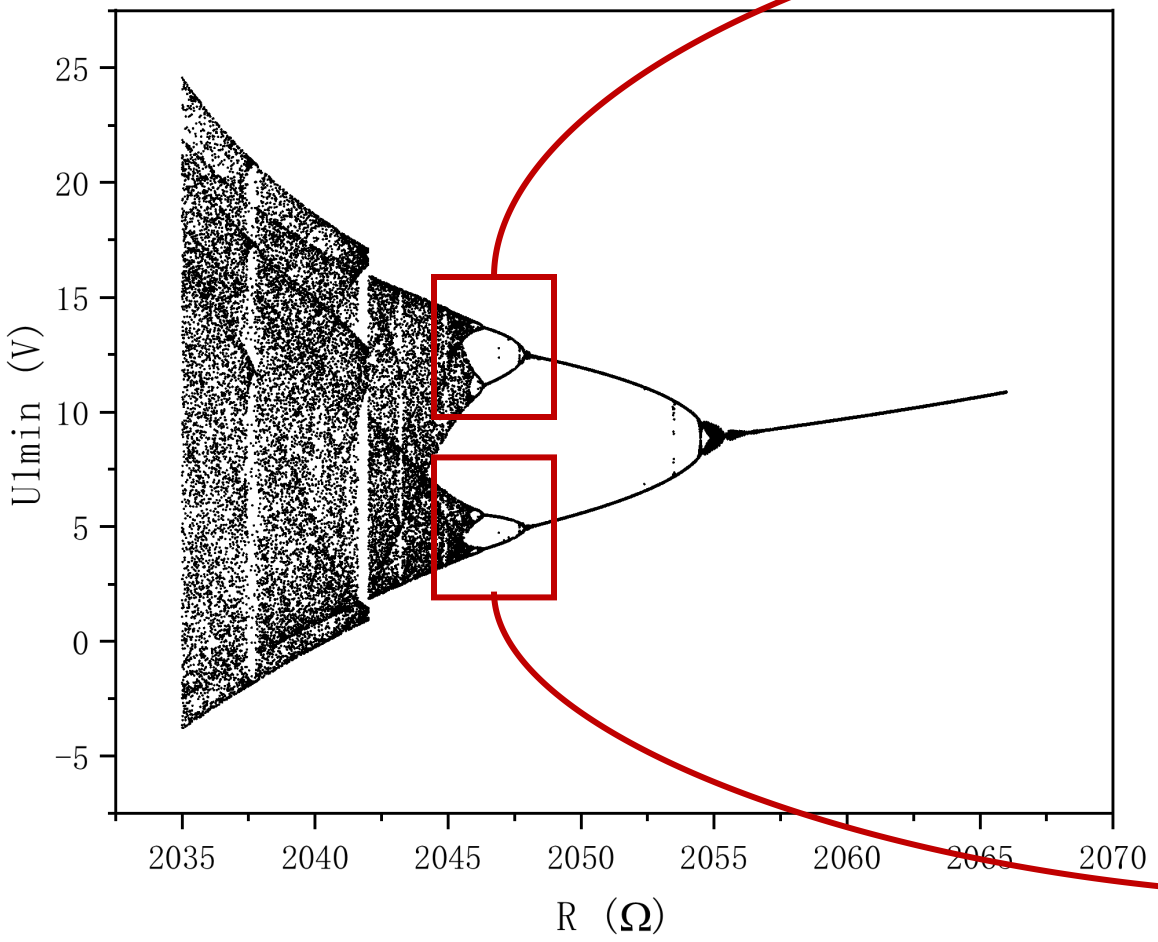


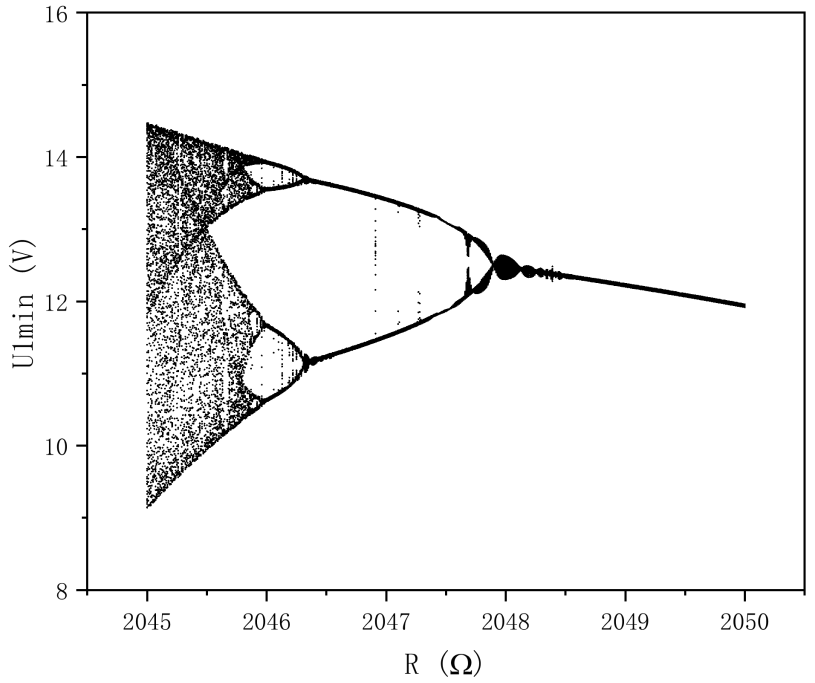
$$\boldsymbol{\delta} = \lim_{n\to\infty}\delta_n = \frac{\mu_n - \mu_{n-1}}{\mu_{n+1} - \mu_n} = 4.6992016091029\ldots$$

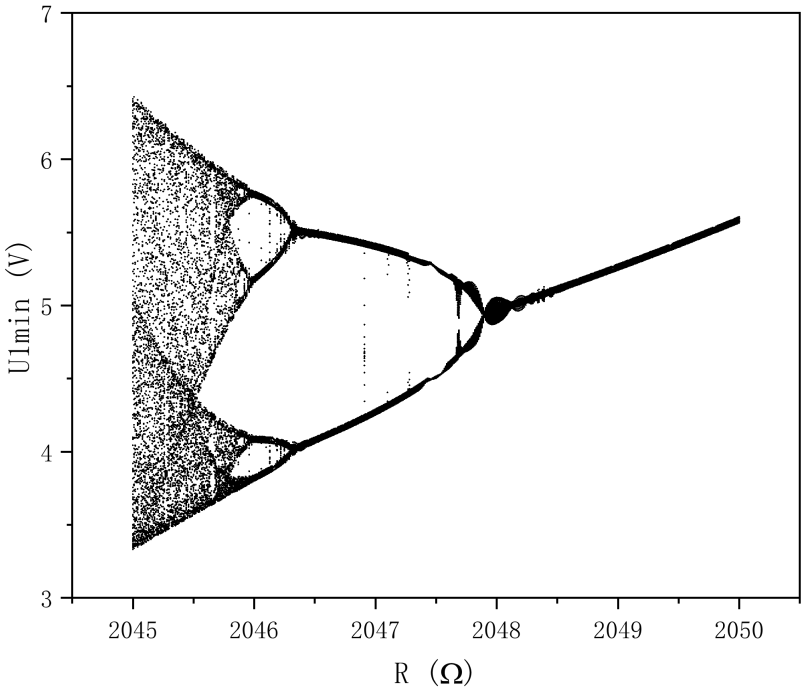
$$\delta_2 = \frac{R_2 - R_1}{R_3 - R_2} = 4.4362 \qquad \text{相对误差：} 5.60\%$$

- **分岔图中的分形**

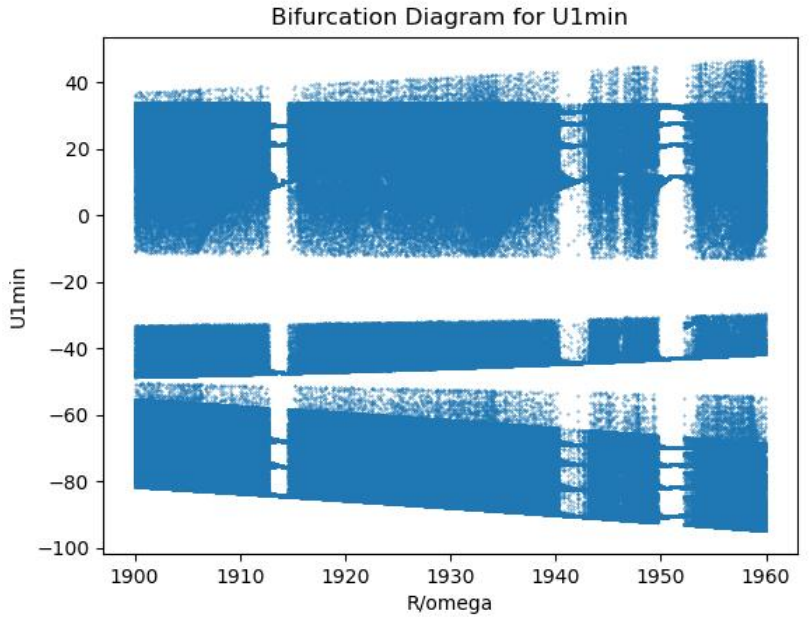
——自相似

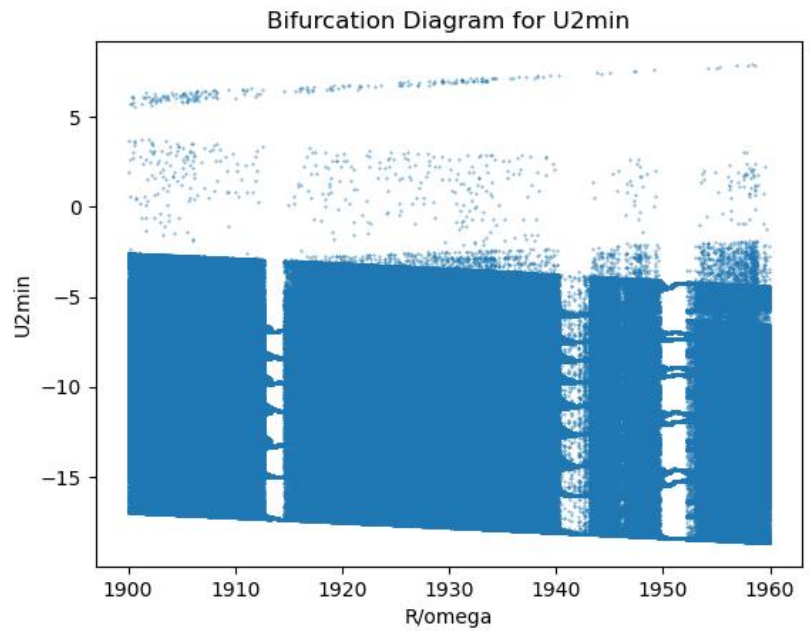

25
20
15
10
5
0
−5
Ulmin (V)
2035
2040
2045
2050
2055
2060
2065
2070
R (Ω)

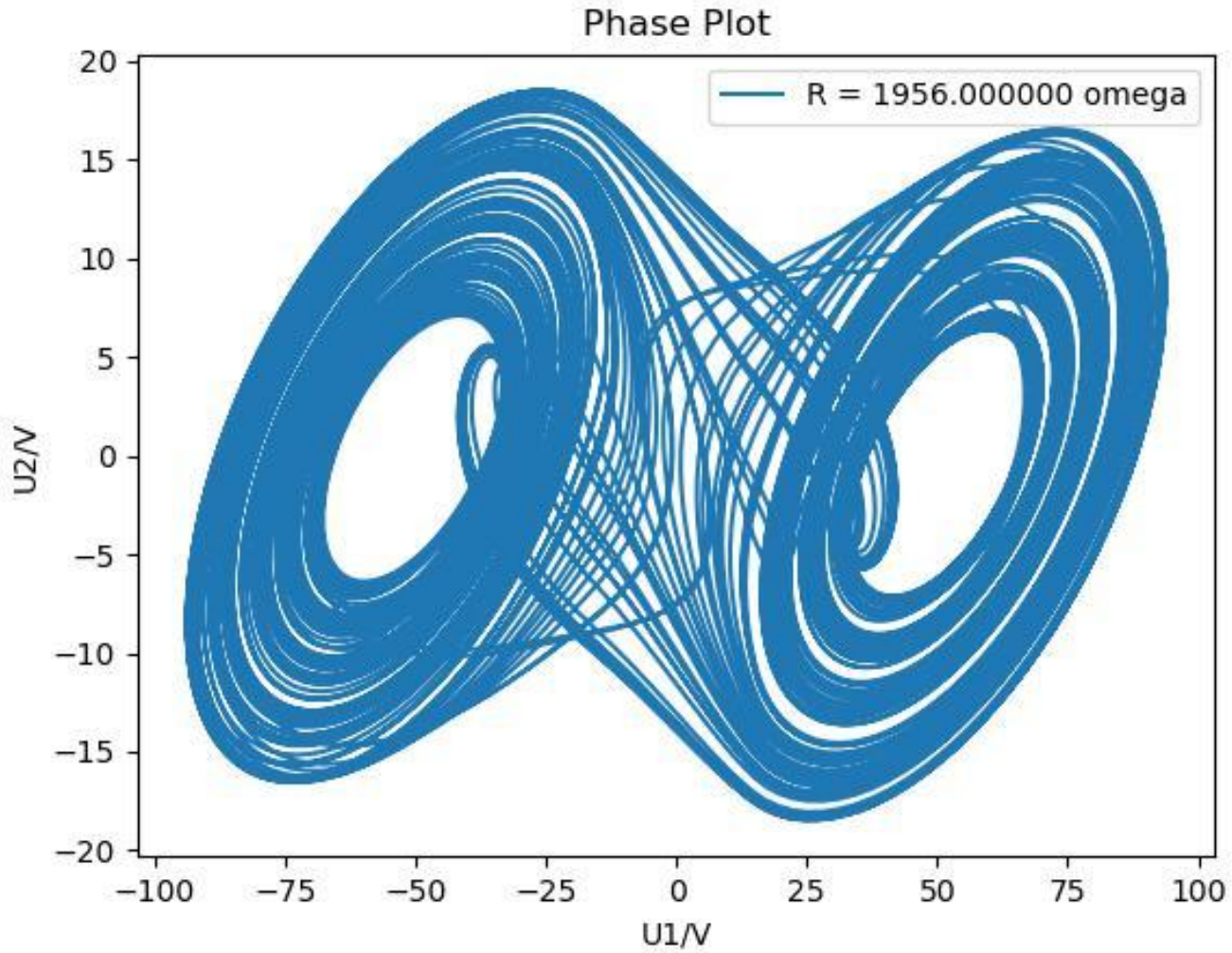

16
14
12
10
8
Ulmin (V)
2045
2046
2047
2048
2049
2050
R (Ω)


7
6
5
4
3
Ulmin (V)
2045
2046
2047
2048
2049
2050
R (Ω)

- 双吸引子的变化规律


Bifurcation Diagram for U1min
U1min
R/omega
40
20
0
−20
−40
−60
−80
−100
1900
1910
1920
1930
1940
1950
1960


Bifurcation Diagram for U2min
U2min
R/omega
5
0
−5
−10
−15
1900
1910
1920
1930
1940
1950
1960


Phase Plot
R = 1956.000000 omega
U2/V
U1/V
20
15
10
5
0
−5
−10
−15
−20
−100
−75
−50
−25
0
25
50
75
100

程序代码

1、相图模拟代码

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# constants
global C1, C2, L, R,G
C1 = 11.061e-9 # F
C2 = 101.0e-9 # F
L = 23.7095e-3 # H
R_start = 1700 #omega
R_end = 2069 #omega
dR = 1

# initial condition
t0 = 0
y0 = np.array([0.0,0.0,0.001]) # [U1, U2, I]
t_final = 0.1
t_total = 20000

# I-V curve of the nonlinear negative resistance
def g(U):
    Ga = -0.00076 # omega^(-1) uncertainty: 0.1*10^(-4)
    Gb = -0.000409 # omega^(-1) uncertainty: 0.06*10^(-4)
    E = 15.0 #V
    g = Gb*U+(Gb-Ga)/2*(abs(U-E)-abs(U+E))
    return g

# chua's circuit
def f(y,t):
    y1 = (G*(y[1]-y[0])-g(y[0]))/C1
    y2 = (G*(y[0]-y[1])+y[2])/C2
    y3 = -y[1]/L
    return np.array([y1, y2, y3])

time=np.linspace(t0,t_final,t_total)
R_list = [R_start + i*dR for i in range(int((R_end-R_start)/dR))]
for R in R_list:
    G = 1/R
    y = odeint(f,y0,time)

    # phase plot
    plt.plot(y[4000:,0],y[4000:,1] , label = 'R = %f omega'%R )
    plt.title('Phase Plot')
    plt.xlabel('U1/V')
    plt.ylabel('U2/V')
    plt.legend()
    plt.savefig('%f omega.jpg'%R)
    plt.clf()
    print('%f omega Done!'%R)
```

2、U-t图及数据模拟代码

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
import pandas as pd

# constants
global C1, C2, L, R,G
C1 = 11.061e-9 # F
C2 = 101.0e-9 # F
L = 23.7095e-3 # H
R_start = 1710 # omega
R_end = 2065 # omega
dR = 1

# initial condition
t0 = 0
y0 = np.array([0.0,0.0,0.001]) # [U1, U2, I]
t_final = 0.1
t_total = 20000

# I-V curve of the nonlinear negative resistance
def g(U):
    Ga = -0.00076 # omega^(-1) uncertainty: 0.1*10^(-4)
    Gb = -0.000409 # omega^(-1) uncertainty: 0.06*10^(-4)
    E = 15.0 #V
    g = Gb*U+(Gb-Ga)/2*(abs(U-E)-abs(U+E))
    return g

# chua's circuit
def f(y,t):
    y1 = (G*(y[1]-y[0])-g(y[0]))/C1
    y2 = (G*(y[0]-y[1])+y[2])/C2
    y3 = -y[1]/L
    return np.array([y1, y2, y3])

time=np.linspace(t0,t_final,t_total)
R_list = [R_start + i*dR for i in range(int((R_end-R_start)/dR))]
for R in R_list:
    G = 1/R
    y = odeint(f,y0,time)

    # simulation data plot
    plt.plot(time[4000:6000:],y[4000:6000,0],label = 'U1') #eliminate unstable
solutions
    plt.plot(time[4000:6000:], y[4000:6000,1], label='U2')
    plt.xlabel('t')
    plt.ylabel('U')
    plt.title('R = %f omega'% round(R, 1))
    plt.legend()
    plt.savefig('%f omega.jpg' % round(R, 1))
    plt.clf()
    print('%f omega Done!' % round(R, 1))

    # output simulating data
```

```
    dataframe = pd.DataFrame({'t': time[4000::].tolist(), 'U1':
y[4000:,0].tolist(),'U2': y[4000:,1].tolist()})
    dataframe.to_csv("%f omega.csv"%round(R,1), index=False, sep=',')
```

3、FFT分析代码

实验数据分析代码如下所示：

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# load data
filename = '20000101_022739s.csv'
# '20000101_021718dd.csv' 双吸引子
#'20000101_021921ss.csv' 单吸引子
#'20000101_022112tr.csv' 三周期
# '20000101_022321zhenfa.csv' 阵发混沌
# '20000101_022614f.csv' 四周期
#'20000101_022700d.csv' 双周期
#'20000101_022739s.csv' 单周期
CH2_raw = pd.read_csv(filename,sep=',',header=9,usecols=[2])
CH1_raw = pd.read_csv(filename,sep=',',header=9,usecols=[1])
array1 = CH1_raw.values
array2 = CH2_raw.values
CH1 = []
CH2 = []
for i in range(len(array1)):
    CH1.append(float(array1[i,0]))
    CH2.append(float(array2[i, 0]))
t = np.arange(len(CH1))

# FFT
CH1_fft = np.fft.rfft(CH1)
power1 = [abs(c) for c in CH1_fft]
CH2_fft = np.fft.rfft(CH2)
power2 = [abs(c) for c in CH2_fft]

# plot the power spectrum
loc = t.tolist().index(1000)
plt.plot(t[:loc],power1[:loc])
plt.xlabel('Frequency')
plt.ylabel('Power')
plt.title('U1 Power Spectrum')
plt.show()

plt.plot(t[:loc],power2[:loc])
plt.xlabel('Frequency')
plt.ylabel('Power')
plt.title('U2 Power Spectrum')
plt.show()
```

模拟数据分析代码在读取文件数据时参数有一些差别，如下所示，其余都与上面分析实验数据的代码一致。

```
CH2_raw = pd.read_csv(filename,sep=',',header=1,usecols=[2])
CH1_raw = pd.read_csv(filename,sep=',',header=1,usecols=[1])
```

4、分岔图代码

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
import pandas as pd

# constants
global C1, C2, L, R,G
C1 = 11.061e-9 # F
C2 = 101.0e-9 # F
L = 23.7095e-3 # H
R_start = 1980 #omega
R_end = 2066 #omega
dR = 0.01

# initial condition
t0 = 0
y0 = np.array([0.0,0.0,0.001]) # [U1, U2, I]
t_final = 0.1
t_total = 20000

# I-V curve of the nonlinear negative resistance
def g(U):
    Ga = -0.00076 # omega^(-1) uncertainty: 0.1*10^(-4)
    Gb = -0.000409 # omega^(-1) uncertainty: 0.06*10^(-4)
    E = 15.0 #V
    g = Gb*U+(Gb-Ga)/2*(abs(U-E)-abs(U+E))
    return g

# chua's circuit
def f(y,t):
    y1 = (G*(y[1]-y[0])-g(y[0]))/C1
    y2 = (G*(y[0]-y[1])+y[2])/C2
    y3 = -y[1]/L
    return np.array([y1, y2, y3])

# find local minimum function
def mini(data):
    '''input the data list'''
    length = len(data)
    ans = []
    for i in range(2,length-2):
        if data[i]<data[i-1] and data[i]<data[i-2] and data[i]<data[i+1] and
data[i]<data[i+2]:
            ans.append(data[i])
    return ans

time=np.linspace(t0,t_final,t_total)
R_list = [R_start + i*dR for i in range(int((R_end-R_start)/dR))]

R1list = [] # for bifurcation diagram
R2list = []
```

```
U1min = [] # also for bifurcation diagram
U2min = []

for R in R_list:
    G = 1/R
    y = odeint(f,y0,time)

    # find the local minimum
    y1list= y[4000:,0].tolist()
    U1min += mini(y1list)
    R1list += [R+i-i for i in range(len(mini(y1list)))]
    y2list = y[4000:, 1].tolist()
    U2min += mini(y2list)
    R2list += [R + i - i for i in range(len(mini(y2list)))]
    print('R = %f omega'%round(R,3))

# plot the bifurcation diagram
plt.scatter(R1list,U1min,s=0.1,alpha = 1)
plt.xlabel('R/omega')
plt.ylabel('U1min')
plt.title('Bifurcation Diagram for U1min')
plt.savefig('U1min')
plt.clf()

plt.scatter(R2list,U2min,s=0.1,alpha = 1)
plt.xlabel('R/omega')
plt.ylabel('U2min')
plt.title('Bifurcation Diagram for U2min')
plt.savefig('U2min')
plt.clf()

# save data
dataframe1 = pd.DataFrame({'R': R1list, 'U1min': U1min})
dataframe1.to_csv("U1min.csv", index=False, sep=',')
dataframe2 = pd.DataFrame({'R': R2list, 'U1min': U2min})
dataframe2.to_csv("U2min.csv", index=False, sep=',')
```